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A STUDY ON OPERATIONS OF BIPOLAR NEUTROSOPHIC CUBIC FUZZY GRAPHS

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ABSTRACT: In this paper we introduce the idea of bipolar neutrosophic cubic fuzzy graphs. We discuss fundamental binary operations like Cartesian product, composition of bipolar neutrosophic cubic fuzzy graphs. We provide some results related with bipolar neutrosophic cubic fuzzy graphs.

INTRODUCTION

In 1975 Rosenfeld [7] introduced fuzzy graphs based on fuzzy set.Fuzzy graph theory plays essential roles in various discipines including information theory, neural networks,clustering problems and control theory,etc.Fuzzy models is more compatiable to the systemin compare with classical mode.Bhattacharya [5] gave some remarks on fuzzy graphs.Some operations on fuzzy graphs were introduced by mordeson and peng[6].Akram et al. has introduced several new concepts including bipolar fuzzy graphs,regular bipolar fuzzy graphs, irregular bipolar fuzzy graphs etc.In this paper, we present certain operations on bipolar fuzzy graphs structures and investigate some of their properties.

1. Basic Definitions

Definition 1.1 Let X be a space of points with generic elements in X denoted by x. A neutrosophic fuzzy set A is characterized by truth-membership function $\mu_{AT}(x)$, an indeterminacy-membership function $\lambda_{AI}(x)$ a falsity – membership function $\gamma_{AF}(x)$.

For each point x in X $\mu_{AT}(x), \lambda_{AI}(x)$, $\gamma_{AF}(x) \in [0,1]$A neutrosophic fuzzy set A can be written as

$$A = \{< x: \mu_{AT}(x), \lambda_{AI}(x), \gamma_{AF}(x) >, x \in X\}$$

Definition 1.2 Let X be a space of points with generic elements in X denoted by x. A neutrosophic cubic fuzzy set in X is a pair $G = (M, N)$ where $M = \{< x: \mu_{MT}(x), \lambda_{MI}(x), \gamma_{MF}(x) >, x \in X\}$ is an interval neutrosophic fuzzy set in X and $N = \{< x: \mu_{NT}(x), \lambda_{NI}(x), \gamma_{NF}(x) >, x \in X\}$ is a neutrosophic fuzzy set in X.

Definition 1.3 Let $G^* = (V, E)$ be a fuzzy graph. By neutrosophic cubic fuzzy graph of G^*, we mean a pair $G = (M, N)$ where $M = (A, B) = \big((\mu_{AT}, \mu_{BT}), (\lambda_{AI}, \lambda_{BI}), (\gamma_{AF}, \gamma_{BF})\big)$ is the neutrosophic cubic fuzzy set representation of vertex set V and $N = (C, D) = \big((\mu_{CT}, \mu_{DT}), (\lambda_{CI}, \lambda_{DI}), (\gamma_{CF}, \gamma_{DF})\big)$ is the neutrosophic cubic fuzzy set representation of edge set E such that

(i) $(\mu_{TC}(x_i y_i) \leq \ r\min\{\mu_{AT}(x_i), \mu_{AT}(y_i)\}, \mu_{DT}(x_i y_i) \leq \max\{\mu_{BT}(x_i), \mu_{BT}(y_i)\})$
(ii) $(\lambda_{IC}(x_i y_i) \leq \ r\min\{\lambda_{AI}(x_i), \lambda_{AI}(y_i)\}, \lambda_{DI}(x_i y_i) \leq \max\{\lambda_{BI}(x_i), \lambda_{BI}(y_i)\})$
(iii) $(\gamma_{FC}(x_i y_i) \leq \ r\min\{\gamma_{AF}(x_i), \gamma_{AF}(y_i)\}, \gamma_{DF}(x_i y_i) \leq \max\{\gamma_{BF}(x_i), \gamma_{BF}(y_i)\})$

2. Bipolar Neutrosophic Cubic Fuzzy Graphs(BNCFG)

Definition 2.1 Let X be a space of points with generic elements in X denoted by x. A Bipolar neutrosophic cubic fuzzy set in X is a pair $G = \big((M^P, N^P), (M^N, N^N)\big)$ is defined as

$$M^P = \{< x^P : \mu^P_{MT}(x), \lambda^P_{MI}(x), \gamma^P_{MF}(x) > / x \in X\}$$

$$M^N = \{< x^N : \mu^N_{MT}(x), \lambda^N_{MI}(x), \gamma^N_{MF}(x) > / x \in X\}$$

is an interval neutrosophic fuzzy set in X and

$$N^P = \{< x^P : \mu^P_{NT}(x), \lambda^P_{NI}(x), \gamma^P_{NF}(x) > / x \in X\}$$

$$N^N = \{< x^N : \mu^N_{NT}(x), \lambda^N_{NI}(x), \gamma^N_{NF}(x) > / x \in X\}$$

is a neutrosophic fuzzy set in X, where $\mu^P_{MT}(x), \lambda^P_{MI}(x), \gamma^P_{MF}(x) \to [0,1]$ and $\mu^N_{MT}(x), \lambda^N_{MI}(x), \gamma^N_{MF}(x) \to [-1,0]$.

Definition 2.2

Let $G^* = (V, E)$ be a fuzzy graph. By a Bipolar neutrosophic cubic fuzzy graph of G^*. We mean a pair $G = \big((M^P, N^P), (M^N, N^N)\big)$ where

$$M^P = (A, B) = \big((\mu^P_{AT}, \mu^P_{BT}), (\lambda^P_{AI}, \lambda^P_{BI}), (\gamma^P_{AF}, \gamma^P_{BF})\big)$$

$$M^N = (A, B) = \big((\mu^N_{AT}, \mu^N_{BT}), (\lambda^N_{AI}, \lambda^N_{BI}), (\gamma^N_{AF}, \gamma^N_{BF})\big)$$

is the neutrosophic cubic fuzzy set representation of vertex set V and

$$N^P = (C, D) = \big((\mu^P_{CT}, \mu^P_{DT}), (\lambda^P_{CI}, \lambda^P_{DI}), (\gamma^P_{CF}, \gamma^P_{DF})\big)$$

$$N^N = (C, D) = \big((\mu^N_{CT}, \mu^N_{DT}), (\lambda^N_{CI}, \lambda^N_{DI}), (\gamma^N_{CF}, \gamma^N_{DF})\big)$$

is the neutrosophic cubic fuzzy set representation of edge set E such that

(i) $(\mu^P_{TC}(x_i y_i) \leq \ r\min\{\mu^P_{AT}(x_i), \mu^P_{AT}(y_i)\}, \mu^P_{DT}(x_i y_i) \leq \max\{\mu^P_{BT}(x_i), \mu^P_{BT}(y_i)\})$
$(\mu^N_{TC}(x_i y_i) \geq \ r\max\{\mu^N_{AT}(x_i), \mu^N_{AT}(y_i)\}, \mu^N_{DT}(x_i y_i) \geq \min\{\mu^N_{BT}(x_i), \mu^N_{BT}(y_i)\})$
(ii) $(\lambda^P_{IC}(x_i y_i) \leq \ r\max\{\lambda^P_{AI}(x_i), \lambda^P_{AI}(y_i)\}, \lambda^P_{DI}(x_i y_i) \leq \min\{\lambda^P_{BI}(x_i), \lambda^P_{BI}(y_i)\})$
$(\lambda^N_{IC}(x_i y_i) \geq \ r\max\{\lambda^N_{AI}(x_i), \lambda^N_{AI}(y_i)\}, \lambda^N_{DI}(x_i y_i) \geq \ \min\{\lambda^N_{BI}(x_i), \lambda^N_{BI}(y_i)\})$
(iii) $(\gamma^P_{FC}(x_i y_i) \leq \ r\max\{\gamma^P_{AF}(x_i), \gamma^P_{AF}(y_i)\}, \gamma^P_{DF}(x_i y_i) \leq \min\{\gamma^P_{BF}(x_i), \gamma^P_{BF}(y_i)\})$
$(\gamma^N_{FC}(x_i y_i) \geq \ r\max\{\gamma^N_{AF}(x_i), \gamma^N_{AF}(y_i)\}, \gamma^N_{DF}(x_i y_i) \geq \min\{\gamma^N_{BF}(x_i), \gamma^N_{BF}(y_i)\})$

3. Operations of Two Bipolar Neutrosophic Cubic Fuzzy Graphs

Definition 3.1 Let $G_1 = \left((M_1{}^P, N_1{}^P), (M_1{}^N, N_1{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = \left((V_1^P, E_1^P), (V_1^N, E_1^N)\right)$ and $G_2 = \left((M_2{}^P, N_2{}^P), (M_2{}^N, N_2{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^* = \left((V_2^P, E_2^P), (V_2^N, E_2^N)\right)$. The Cartesian product of G_1 and G_2 is denoted by

$$G_1 \times G_2 = \left(\left((M_1{}^P \times M_2{}^P), (M_1{}^N \times M_2{}^N)\right), \left((N_1{}^P \times N_2{}^P), (N_1{}^N \times N_2{}^N)\right)\right)$$

$$= \left(\left((A_1^P, B_1^P), (A_1^N, B_1^N)\right) \times \left((A_2^P, B_2^P), (A_2^N, B_2^N)\right), \left((C_1^P, D_1^P), (C_1^N, D_1^N)\right) \times \left((C_2^P, D_2^P), (C_2^N, D_2^N)\right)\right)$$

$$\left\{\begin{pmatrix} \left((\tilde{\mu}^P_{TA_1\times TA_2}, \tilde{\mu}^N_{TA_1\times TA_2}), (\tilde{\mu}^P_{TB_1\times TB_2}, \tilde{\mu}^N_{TB_1\times TB_2})\right), \left((\tilde{\lambda}^P_{IA_1\times IA_2}, \tilde{\lambda}^N_{IA_1\times IA_2}), (\tilde{\lambda}^P_{IB_1\times IB_2}, \tilde{\lambda}^N_{IB_1\times IB_2})\right), \\ \left((\tilde{\gamma}^P_{FA_1\times FA_2}, \tilde{\gamma}^N_{FA_1\times FA_2}), (\tilde{\gamma}^P_{FB_1\times FB_2}, \tilde{\gamma}^N_{FB_1\times FB_2})\right) \end{pmatrix}\right.$$

$$\left.\begin{pmatrix} \left((\tilde{\mu}^P_{TC_1\times TC_2}, \tilde{\mu}^N_{TC_1\times TC_2}), (\tilde{\mu}^P_{TD_1\times TD_2}, \tilde{\mu}^N_{TD_1\times TD_2})\right), \left((\tilde{\lambda}^P_{IC_1\times IC_2}, \tilde{\lambda}^N_{IC_1\times IC_2}), (\tilde{\lambda}^P_{ID_1\times ID_2}, \tilde{\lambda}^N_{ID_1\times ID_2})\right), \\ \left((\tilde{\gamma}^P_{FC_1\times FC_2}, \tilde{\gamma}^N_{FC_1\times FC_2}), (\tilde{\gamma}^P_{FD_1\times FD_2}, \tilde{\gamma}^N_{FD_1\times FD_2})\right) \end{pmatrix}\right\}$$

and is defined as follows

(i) $$\begin{pmatrix} \left(\tilde{\mu}^P_{TA_1\times TA_2}(u,v) = r^P \min\left(\tilde{\mu}^P_{TA_1}(u), \tilde{\mu}^P_{TA_2}(v)\right), \tilde{\mu}^P_{TB_1\times TB_2}(u,v) = \max\left(\tilde{\mu}^P_{TB_1}(u), \tilde{\mu}^P_{TB_2}(v)\right)\right), \\ \left(\tilde{\mu}^N_{TA_1\times TA_2}(u,v) = r^N \max\left(\tilde{\mu}^N_{TA_1}(u), \tilde{\mu}^N_{TA_2}(v)\right), \tilde{\mu}^N_{TB_1\times TB_2}(u,v) = \min\left(\tilde{\mu}^N_{TB_1}(u), \tilde{\mu}^N_{TB_2}(v)\right)\right) \end{pmatrix}$$

(ii) $$\begin{pmatrix} \left(\tilde{\lambda}^P_{IA_1\times IA_2}(u,v) = r^P \min\left(\tilde{\lambda}^P_{IA_1}(u), \tilde{\lambda}^P_{IA_2}(v)\right), \tilde{\lambda}^P_{IB_1\times IB_2}(u,v) = \max\left(\tilde{\lambda}^P_{IB_1}(u), \tilde{\lambda}^P_{IB_2}(v)\right)\right), \\ \left(\tilde{\lambda}^N_{IA_1\times IA_2}(u,v) = r^N \max\left(\tilde{\lambda}^N_{IA_1}(u), \tilde{\lambda}^N_{IA_2}(v)\right), \tilde{\lambda}^N_{IB_1\times IB_2}(u,v) = \min\left(\tilde{\lambda}^N_{IB_1}(u), \tilde{\lambda}^N_{IB_2}(v)\right)\right) \end{pmatrix}$$

(iii) $$\begin{pmatrix} \left(\tilde{\gamma}^P_{FA_1\times FA_2}(u,v) = r^P \max\left(\tilde{\gamma}^P_{FA_1}(u), \tilde{\gamma}^P_{FA_2}(v)\right), \tilde{\gamma}^P_{FB_1\times FB_2}(u,v) = \min\left(\tilde{\gamma}^P_{FB_1}(u), \tilde{\gamma}^P_{FB_2}(v)\right)\right), \\ \left(\tilde{\gamma}^N_{FA_1\times FA_2}(u,v) = r^N \min\left(\tilde{\gamma}^N_{FA_1}(u), \tilde{\gamma}^N_{FA_2}(v)\right), \tilde{\gamma}^N_{FB_1\times FB_2}(u,v) = \max\left(\tilde{\gamma}^N_{FB_1}(u), \tilde{\gamma}^N_{FB_2}(v)\right)\right) \end{pmatrix}$$

(iv) $\left(\left(\left(\tilde{\mu}^P_{TC_1\times TC_2}((u,v_1)(u,v_2)) = r^P \, min\left(\tilde{\mu}^P_{TA_1}(u), \tilde{\mu}^P_{TC_2}(v_1 v_2)\right)\right), \left(\tilde{\mu}^N_{TC_1\times TC_2}((u,v_1)(u,v_2)) =\right.\right.\right.$

rNmaxμTA1Nu, μTC2Nv1v2, μTD1×TD2Pu,v1u,v2=maxμTB1Pu, μTD2Pv1v2, μTD1×TD2Nu,v1u,v2=minμTB1Nu, μTD2Nv1v2

$\forall v \in V_2$ and $u_1 u_2 \in E_1$

(v) $\left(\left(\left(\tilde{\lambda}^P_{IC_1\times IC_2}((u,v_1)(u,v_2)) = r^P \, min\left(\tilde{\lambda}^P_{IA_1}(u), \tilde{\lambda}^P_{IC_2}(v_1 v_2)\right)\right), \left(\tilde{\lambda}^N_{IC_1\times IC_2}((u,v_1)(u,v_2)) =\right.\right.\right.$

rNmaxλIA1Nu, λIC2Nv1v2, λID1×ΙD2Nu,v1u,v2=minλIB1Nu, λID2Nv1v2, λID1×ID2Pu,v1u,v2=maxλIB1Pu, λID2Pv1v2, λID1×ID2Nu,v1u,v2=minλIB1Nu, λID2Nv1v2

$\forall v \in V_2$ and $u_1 u_2 \in E_1$

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(vi) $\left(\left(\left(\tilde{\gamma}^P_{FC_1\times FC_2}((u,v_1)(u,v_2)) = r^P \max\left(\tilde{\gamma}^P_{FA_1}(u), \tilde{\gamma}^P_{FC_2}(v_1v_2)\right)\right), \left(\tilde{\gamma}^N_{FC_1\times FC_2}((u,v_1)(u,v_2)) =\right.\right.\right.$

*rN*min*γFA1Nu, γFC2Nv1v2, γFD1×FD2Pu,v1u,v2*=min*γFB1Pu, γFD2Pv1v2, γFD1×FD2Nu,v1u,v2*=max*γFB1Nu, γFD2Nv1v2*

$\forall v \in V_2$and $u_1u_2 \in E_1$

(vii) $\left(\left(\left(\tilde{\mu}^P_{TC_1\times TC_2}((u_1,v)(u_2,v)) = r^P \min\left(\tilde{\mu}^P_{TC_1}(u_1u_2), \tilde{\mu}^P_{TA_2}(v)\right)\right), \left(\tilde{\mu}^N_{TC_1\times TC_2}((u_1,v)(u_2,v)) =\right.\right.\right.$

*rN*max*μTC1Nu1u2, μTA2Nv, μTD1×TD2Pu1,vu2,v*=max*μTD1Pu1u2, μTBPv, μTD1×TD2Nu1,vu2,v*=min*μTD1Nu1u2, μTB2Nv*

(viii) $\left(\left(\left(\tilde{\lambda}^P_{IC_1\times IC_2}((u_1,v)(u_2,v)) = r^P \min\left(\tilde{\lambda}^P_{IC_1}(u_1u_2), \tilde{\lambda}^P_{IA_2}(v)\right)\right), \left(\tilde{\lambda}^N_{IC_1\times IC_2}((u_1,v)(u_2,v)) =\right.\right.\right.$

*rN*max*λIC1Nu1u2, λIA2Nv, λID1×ID2Pu1,vu2,v*=max*λID1Pu1u2, λIB2Pv, λID1×ID2Nu1,vu2,v*=min*λID1Nu1u2, λIB2Nv*

(ix) $\left(\left(\left(\tilde{\gamma}^P_{FC_1\times FC_2}((u_1,v)(u_2,v)) = r^P \max\left(\tilde{\gamma}^P_{FC_1}(u_1u_2), \tilde{\gamma}^P_{FA_2}(v)\right)\right), \left(\tilde{\gamma}^N_{FC_1\times FC_2}((u_1,v)(u_2,v)) =\right.\right.\right.$

*rN*min*γFC1Nu1u2, γFA2Nv, γFD1×FD2Pu1,vu2,v*=min*γFD1Pu1u2, γFB2Pv, γFD1×FD2Nu1,vu2,v*=max*γFD1Nu1u2, γFB2Nv*

$\forall (u,v) \in (V_1, V_2)$

Example 3.2

Let $G_1 = \left((M_1{}^P, N_1{}^P), (M_1{}^N, N_1{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = (V_1, E_1)$ where $v_1 = \{u, v, w\}, E = \{uv, vw, uw\}$

$$\{u, ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.5, 0.6], 0.1)\}$$

$$M_1{}^P = \langle\{v, ([0.1, 0.3], 0.1), ([0.4, 0.5], 0.3), ([0.1, 0.1], 0.2)\}\rangle$$

$$\{w, ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\}$$

$$\{uv,([0.1,0.1],0.4),([0.3,0.4],0.2),([0.5,0.6],0.1)\}$$

$$N_1{}^P=\langle\{vw,([0.1,0.3],0.1),([0.1,0.2],0.6),([0.3,0.5],0.2)\}\rangle$$

$$\{uw,([0.2,0.3],0.1),([0.1,0.2],0.6),([0.5,0.6],0.1)\}$$

$$\{u,([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.5,-0.6],-0.1)\}$$

$$M_1{}^N=\langle\{v,([-0.1,-0.3],-0.1),([-0.4,-0.5],-0.3),([-0.1,-0.1],-0.2)\}\rangle$$

$$\{w,([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\}$$

$$\{uv,([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.5,-0.6],-0.1)\}$$

$$N_1{}^N=\langle\{vw,([-0.1,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\}\rangle$$

$$\{uw,([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.1)\}$$

and $G_2=\left((M_2{}^P,N_2{}^P),(M_2{}^N,N_2{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^*=(V_2,E_2)$ where $V_1=\{a,v,c\}$ and $E_2=\{ab,bc,ac\}$

$$\{a,([0.6,0.7],0.5),([0.1,0.3],0.4),([0.2,0.3],0.6)\}$$

$$M_2{}^P=\langle\{b,([0.1,0.2],0.3),([0.5,0.6],0.2),([0.8,0.9],0.4)\}\rangle$$

$$\{c,([0.3,0.4],0.1),([0.2,0.3],0.1),([0.5,0.6],0.3)\}$$

$$\{ab,([0.1,0.2],0.5),([0.1,0.3],0.4),([0.8,0.9],0.4)\}$$

$$N_2{}^P=\langle\{bc,([0.1,0.2],0.3),([0.2,0.3],0.2),([0.8,0.9],0.3)\}\rangle$$

$$\{ac,([0.3,0.4],0.5),([0.1,0.3],0.4),([0.5,0.6],0.3)\}$$

$$\{a,([-0.6,-0.7],-0.5),([-0.1,-0.3],-0.4),([-0.2,-0.3],-0.6)\}$$

$$M_2{}^N=\langle\{b,([-0.1,-0.2],-0.3),([-0.5,-0.6],-0.2),([-0.8,-0.9],-0.4)\}\rangle$$

$$\{c,([-0.3,-0.4],-0.1),([-0.2,-0.3],-0.1),([-0.5,-0.6],-0.3)\}$$

$$\{ab,([-0.1,-0.2],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.4)\}$$

$$N_2{}^N=\langle\{bc,([-0.1,-0.2],-0.3),([-0.2,-0.3],-0.2),([-0.8,-0.9],-0.3)\}\rangle$$

$$\{ac,([-0.3,-0.4],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.3)\}$$

then $G_1\times G_2$ is a bipolar neutrosophic cubic fuzzy graph of $G_1^*\times G_2^*$,

where $V_1\times V_2=\{(u,a),(u,b),(u,c),(v,a),(v,b),(v,c),(w,a),(w,b),(w,c)\}$ and

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$$M_1^P \times M_2^P = \left\langle \begin{array}{c} \{(u,a),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.5,0.6],0.1)\} \\ \{(u,b),([0.1,0.1],0.4),([0.3,0.4],0.2),([0.8,0.9],0.1)\} \\ \{(u,c),([0.1,0.1],0.6),([0.2,0.3],0.2),([0.5,0.6],0.1)\} \\ \{(v,a),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.2,0.3],0.2)\} \\ \{(v,b),([0.1,0.2],0.3),([0.4,0.5],0.3),([0.8,0.9],0.2)\} \\ \{(v,c),([0.1,0.3],0.1),([0.2,0.3],0.3),([0.5,0.6],0.2)\} \\ \{(w,a),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.3,0.5],0.2)\} \\ \{(w,b),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{(w,c),([0.2,0.3],0.1),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \end{array} \right\rangle$$

$$M_1^N \times M_2^N = \left\langle \begin{array}{c} \{(u,a),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.1)\} \\ \{(u,b),([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.8,-0.9],-0.1)\} \\ \{(u,c),([-0.1,-0.1],-0.6),([-0.2,-0.3],-0.2),([-0.5,-0.6],-0.1)\} \\ \{(v,a),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.2,-0.3],-0.2)\} \\ \{(v,b),([-0.1,-0.2],-0.3),([-0.4,-0.5],-0.3),([-0.8,-0.9],-0.2)\} \\ \{(v,c),([-0.1,-0.3],-0.1),([-0.2,-0.3],-0.3),([-0.5,-0.6],-0.2)\} \\ \{(w,a),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\} \\ \{(w,b),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{(w,c),([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \end{array} \right\rangle$$

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$$N_1^P \times N_2^P = \left\langle \begin{array}{l} \{((u,a),(u,b)),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.8,0.9],0.1)\} \\ \{((u,b),(u,c)),([0.1,0.1],0.4),([0.2,0.3],0.2),([0.8,0.9],0.1)\} \\ \{((u,a),(v,c)),([0.1,0.1],0.4),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,c)),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,b)),([0.1,0.2],0.5),([0.1,0.3],0.4),([0.8,0.9],0.2)\} \\ \{((v,b),(w,b)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,b),(w,c)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,a),(w,c)),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \\ \{((u,ab),(w,a)),([0.1,0.1],0.5),([0.1,0.2],0.6),([0.5,0.6],0.1)\} \end{array} \right\rangle$$

$$N_1^N \times N_2^N = \left\langle \begin{array}{l} \{((u,a),(u,b)),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.1)\} \\ \{((u,b),(u,c)),([-0.1,-0.1],-0.4),([-0.2,-0.3],-0.2),([-0.8,-0.9],-0.1)\} \\ \{((u,a),(v,c)),([-0.1,-0.1],-0.4),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,c)),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,b)),([-0.1,-0.2],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.2)\} \\ \{((v,b),(w,b)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,b),(w,c)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,a),(w,c)),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \\ \{((u,ab),(w,a)),([-0.1,-0.1],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.1)\} \end{array} \right\rangle$$

Definition 3.3 Let $G_1 = \left((M_1{}^P, N_1{}^P),(M_1{}^N, N_1{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = (V_1, E_1)$ and $G_2 = \left((M_2{}^P, N_2{}^P),(M_2{}^N, N_2{}^N)\right)$ be a Bipolar neutrosophic cubic fuzzy graph of $G_2^* = (V_2, E_2)$. Then composition of G_1 and G_2 is denoted by $G_1[G_2]$ and defined as follows

$$G_1[G_2] = \left((M_1{}^P, N_1{}^P),(M_1{}^N, N_1{}^N)\right)\left[(M_2{}^P, N_2{}^P),(M_2{}^N, N_2{}^N)\right]$$

$$= \{(M_1^P, M_1^N)[M_2^P, M_2^N],(N_1^P, N_1^N)[N_2^P, N_2^N]\}$$

$$= \left\{ \begin{array}{l} \left((A_1^P, A_1^N),(B_1^P, B_1^N)\right)\left[\left((A_2^P, A_2^N),(B_2^P, B_2^N)\right)\right] \\ \left((C_1^P, D_1^N),(C_1^P, D_1^N)\right)\left[\left((C_2^P, D_2^N),(C_2^P, D_2^N)\right)\right] \end{array} \right\}$$

$$= \begin{Bmatrix} (A_1^P, A_1^N), [A_2^P, A_2^N], (B_1^P, B_1^N), [B_2^P, B_2^N] \\ (C_1^P, C_1^N), [C_2^P, C_2^N], (D_1^P, D_1^N), [D_2^P, D_2^N] \end{Bmatrix}$$

$$= \begin{Bmatrix} < \left((\tilde{\mu}_{TA_1}^P, \tilde{\mu}_{TA_1}^N) \circ (\tilde{\mu}_{TA_2}^P, \tilde{\mu}_{TA_2}^N)\right), \left((\tilde{\mu}_{TB_1}^P, \tilde{\mu}_{TB_1}^N) \circ (\tilde{\mu}_{TB_2}^P, \tilde{\mu}_{TB_2}^N)\right), \\ \left((\tilde{\lambda}_{IA_1}^P, \tilde{\lambda}_{IA_1}^N) \circ (\tilde{\lambda}_{IA_2}^P, \tilde{\lambda}_{IA_2}^N)\right), \left((\tilde{\lambda}_{IB_1}^P, \tilde{\lambda}_{IB_1}^N) \circ (\tilde{\lambda}_{IB_2}^P, \tilde{\lambda}_{IB_2}^N)\right), \\ \left((\tilde{\gamma}_{FA_1}^P, \tilde{\gamma}_{FA_1}^N) \circ (\tilde{\gamma}_{FA_2}^P, \tilde{\gamma}_{FA_2}^N)\right), \left((\tilde{\gamma}_{FB_1}^P, \tilde{\gamma}_{FB_1}^N) \circ (\tilde{\gamma}_{FB_2}^P, \tilde{\gamma}_{FB_2}^N)\right) > \\ < \left((\tilde{\mu}_{TC_1}^P, \tilde{\mu}_{TC_1}^N) \circ (\tilde{\mu}_{TC_2}^P, \tilde{\mu}_{TC_2}^N)\right), \left((\tilde{\mu}_{TD_1}^P, \tilde{\mu}_{TD_1}^N) \circ (\tilde{\mu}_{TD_2}^P, \tilde{\mu}_{TD_2}^N)\right), \\ \left((\tilde{\lambda}_{IC_1}^P, \tilde{\lambda}_{IC_1}^N) \circ (\tilde{\lambda}_{IC_2}^P, \tilde{\lambda}_{IC_2}^N)\right), \left((\tilde{\lambda}_{ID_1}^P, \tilde{\lambda}_{ID_1}^N) \circ (\tilde{\lambda}_{ID_2}^P, \tilde{\lambda}_{ID_2}^N)\right), \\ \left((\tilde{\gamma}_{FC_1}^P, \tilde{\gamma}_{FC_1}^N) \circ (\tilde{\gamma}_{FC_2}^P, \tilde{\gamma}_{FC_2}^N)\right), \left((\tilde{\gamma}_{FD_1}^P, \tilde{\gamma}_{FD_1}^N) \circ (\tilde{\gamma}_{FD_2}^P, \tilde{\gamma}_{FD_2}^N)\right) > \end{Bmatrix}$$

where (i) $\forall\left((u^P, u^N)(v^P, v^N)\right) \in (V_1, V_2)$

$$\begin{Bmatrix} (\tilde{\mu}_{TA_1}^P \circ \tilde{\mu}_{TA_2}^P)(u^P, v^P) = r^P \min\left(\tilde{\mu}_{TA_1}^P(u^P), \tilde{\mu}_{TA_2}^P(v^P)\right), \\ (\tilde{\mu}_{TB_1}^P \circ \tilde{\mu}_{TB_2}^P)(u^P, v^P) = \max\left(\tilde{\mu}_{TB_1}^P(u^P), \tilde{\mu}_{TB_2}^P(v^P)\right) \\ (\tilde{\mu}_{TA_1}^N \circ \tilde{\mu}_{TA_2}^N)(u^N, v^N) = r^N \max\left(\tilde{\mu}_{TA_1}^N(u^N), \tilde{\mu}_{TA_2}^N(v^N)\right), \\ (\tilde{\mu}_{TB_1}^N \circ \tilde{\mu}_{TB_2}^N)(u^N, v^N) = \min\left(\tilde{\mu}_{TB_1}^N(u^N), \tilde{\mu}_{TB_2}^N(v^N)\right) \end{Bmatrix}$$

$$\begin{Bmatrix} (\tilde{\lambda}_{IA_1}^P \circ \tilde{\lambda}_{IA_2}^P)(u^P, v^P) = r^P \min\left(\tilde{\lambda}_{IA_1}^P(u^P), \tilde{\lambda}_{IA_2}^P(v^P)\right), \\ (\tilde{\lambda}_{IB_1}^P \circ \tilde{\lambda}_{IB_2}^P)(u^P, v^P) = \max\left(\tilde{\lambda}_{IB_1}^P(u^P), \tilde{\lambda}_{IB_2}^P(v^P)\right) \\ (\tilde{\lambda}_{IA_1}^N \circ \tilde{\lambda}_{IA_2}^N)(u^N, v^N) = r^N \max\left(\tilde{\lambda}_{IA_1}^N(u^N), \tilde{\lambda}_{IA_2}^N(v^N)\right), \\ (\tilde{\lambda}_{IB_1}^N \circ \tilde{\lambda}_{IB_2}^N)(u^N, v^N) = \min\left(\tilde{\lambda}_{IB_1}^N(u^N), \tilde{\lambda}_{IB_2}^N(v^N)\right) \end{Bmatrix}$$

$$\begin{Bmatrix} (\tilde{\gamma}_{FA_1}^P \circ \tilde{\gamma}_{FA_2}^P)(u^P, v^P) = r^P \max\left(\tilde{\gamma}_{FA_1}^P(u^P), \tilde{\gamma}_{FA_2}^P(v^P)\right), \\ (\tilde{\gamma}_{FB_1}^P \circ \tilde{\gamma}_{FB_2}^P)(u^P, v^P) = \min\left(\tilde{\gamma}_{FB_1}^P(u^P), \tilde{\gamma}_{FB_2}^P(v^P)\right) \\ (\tilde{\gamma}_{FA_1}^N \circ \tilde{\gamma}_{FA_2}^N)(u^N, v^N) = r^N \min\left(\tilde{\gamma}_{FA_1}^N(u^N), \tilde{\gamma}_{FA_2}^N(v^N)\right), \\ (\tilde{\gamma}_{FB_1}^N \circ \tilde{\gamma}_{FB_2}^N)(u^N, v^N) = \max\left(\tilde{\gamma}_{FB_1}^N(u^N), \tilde{\gamma}_{FB_2}^N(v^N)\right) \end{Bmatrix}$$

(ii) $\forall (u^P, u^N) \in V_1$ and $(v_1^P v_2^P)(v_1^N v_2^N) \in E$

$$\begin{Bmatrix} (\tilde{\mu}_{TC_1}^P \circ \tilde{\mu}_{TC_2}^P)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = r^P \min\left(\tilde{\mu}_{TC_1}^P(u^P), \tilde{\mu}_{TC_2}^P(v_1^P v_2^P)\right), \\ (\tilde{\mu}_{TD_1}^P \circ \tilde{\mu}_{TD_2}^P)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = \max\left(\tilde{\mu}_{TD_1}^P(u^P), \tilde{\mu}_{TD_2}^P(v_1^P v_2^P)\right) \\ (\tilde{\mu}_{TC_1}^N \circ \tilde{\mu}_{TC_2}^N)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = r^N \max\left(\tilde{\mu}_{TC_1}^N(u^N), \tilde{\mu}_{TC_2}^N(v_1^N v_2^N)\right), \\ (\tilde{\mu}_{TD_1}^N \circ \tilde{\mu}_{TD_2}^N)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = \min\left(\tilde{\mu}_{TD_1}^N(u^N), \tilde{\mu}_{TD_2}^N(v_1^N v_2^N)\right) \end{Bmatrix}$$

$$\begin{cases} \left(\tilde{\lambda}^P_{IC_1} \circ \tilde{\lambda}^P_{IC_2}\right)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = r^P \min\left(\tilde{\lambda}^P_{IC_1}(u^P), \tilde{\lambda}^P_{IC_2}(v_1^P v_2^P)\right), \\ \left(\tilde{\lambda}^P_{ID_1} \circ \tilde{\lambda}^P_{ID_2}\right)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = \max\left(\tilde{\lambda}^P_{ID_1}(u^P), \tilde{\lambda}^P_{ID_2}(v_1^P v_2^P)\right) \\ \left(\tilde{\lambda}^N_{IC_1} \circ \tilde{\lambda}^N_{IC_2}\right)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = r^N \max\left(\tilde{\lambda}^N_{IC_1}(u^N), \tilde{\lambda}^N_{IC_2}(v_1^N v_2^N)\right), \\ \left(\tilde{\lambda}^N_{ID_1} \circ \tilde{\lambda}^N_{ID_2}\right)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = \min\left(\tilde{\lambda}^N_{ID_1}(u^N), \tilde{\lambda}^N_{ID_2}(v_1^N v_2^N)\right) \end{cases}$$

$$\begin{cases} \left(\tilde{\gamma}^P_{FC_1} \circ \tilde{\gamma}^P_{FC_2}\right)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = r^P \max\left(\tilde{\gamma}^P_{FC_1}(u^P), \tilde{\gamma}^P_{FC_2}(v_1^P v_2^P)\right), \\ \left(\tilde{\gamma}^P_{FD_1} \circ \tilde{\gamma}^P_{FD_2}\right)\left((u^P, {v_1}^P)(u^P, {v_2}^P)\right) = \min\left(\tilde{\gamma}^P_{FD_1}(u^P), \tilde{\gamma}^P_{FD_2}(v_1^P v_2^P)\right) \\ \left(\tilde{\gamma}^N_{FC_1} \circ \tilde{\gamma}^N_{FC_2}\right)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = r^N \min\left(\tilde{\gamma}^N_{FC_1}(u^N), \tilde{\gamma}^N_{FC_2}(v_1^N v_2^N)\right), \\ \left(\tilde{\gamma}^N_{FD_1} \circ \tilde{\gamma}^N_{FD_2}\right)\left((u^N, {v_1}^N)(u^N, {v_2}^N)\right) = \max\left(\tilde{\gamma}^N_{FD_1}(u^N), \tilde{\gamma}^N_{FD_2}(v_1^N v_2^N)\right) \end{cases}$$

(iii) $\forall (v^P, v^N) \in V_1$ and $(u_1^P u_2^P)(u_1^N u_2^N) \in E_1$

$$\begin{cases} \left(\tilde{\mu}^P_{TC_1} \circ \tilde{\mu}^P_{TC_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = r^P \min\left(\tilde{\mu}^P_{TC_1}(u_1^P u_2^P), \tilde{\mu}^P_{TA_2}(v^P)\right), \\ \left(\tilde{\mu}^P_{TD_1} \circ \tilde{\mu}^P_{TD_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = \max\left(\tilde{\mu}^P_{TD_1}(u_1^P u_2^P), \tilde{\mu}^P_{TB_2}(v^P)\right) \\ \left(\tilde{\mu}^N_{TC_1} \circ \tilde{\mu}^N_{TC_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = r^N \max\left(\tilde{\mu}^N_{TC_1}(u_1^N u_2^N), \tilde{\mu}^N_{TA_2}(v^N)\right), \\ \left(\tilde{\mu}^N_{TD_1} \circ \tilde{\mu}^N_{TD_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = \min\left(\tilde{\mu}^N_{TD_1}(u_1^N u_2^N), \tilde{\mu}^N_{TB_2}(v^N)\right) \end{cases}$$

$$\begin{cases} \left(\tilde{\lambda}^P_{IC_1} \circ \tilde{\lambda}^P_{IC_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = r^P \min\left(\tilde{\lambda}^P_{IC_1}(u_1^P u_2^P), \tilde{\lambda}^P_{IA_2}(v^P)\right), \\ \left(\tilde{\lambda}^P_{ID_1} \circ \tilde{\lambda}^P_{ID_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = \max\left(\tilde{\lambda}^P_{ID_1}(u_1^P u_2^P), \tilde{\lambda}^P_{IB_2}(v^P)\right) \\ \left(\tilde{\lambda}^N_{IC_1} \circ \tilde{\lambda}^N_{IC_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = r^N \max\left(\tilde{\lambda}^N_{IC_1}(u_1^N u_2^N), \tilde{\lambda}^N_{IA_2}(v^N)\right), \\ \left(\tilde{\lambda}^N_{ID_1} \circ \tilde{\lambda}^N_{ID_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = \min\left(\tilde{\lambda}^N_{ID_1}(u_1^N u_2^N), \tilde{\lambda}^N_{IB_2}(v^N)\right) \end{cases}$$

$$\begin{cases} \left(\tilde{\gamma}^P_{FC_1} \circ \tilde{\gamma}^P_{FC_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = r^P \max\left(\tilde{\gamma}^P_{FC_1}(u_1^P u_2^P), \tilde{\gamma}^P_{FA_2}(v^P)\right), \\ \left(\tilde{\gamma}^P_{FD_1} \circ \tilde{\gamma}^P_{FD_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = \min\left(\tilde{\gamma}^P_{FD_1}(u_1^P u_2^P), \tilde{\gamma}^P_{FB_2}(v^P)\right) \\ \left(\tilde{\gamma}^N_{FC_1} \circ \tilde{\gamma}^N_{FC_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = r^N \min\left(\tilde{\gamma}^N_{FC_1}(u_1^N u_2^N), \tilde{\gamma}^N_{FA_2}(v^N)\right), \\ \left(\tilde{\gamma}^N_{FD_1} \circ \tilde{\gamma}^N_{FD_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = \max\left(\tilde{\gamma}^N_{FD_1}(u_1^N u_2^N), \tilde{\gamma}^N_{FB_2}(v^N)\right) \end{cases}$$

$$\begin{cases} \left(\tilde{\lambda}^P_{IC_1} \circ \tilde{\lambda}^P_{IC_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = r^P \min\left(\tilde{\lambda}^P_{IC_1}(u_1^P u_2^P), \tilde{\lambda}^P_{IA_2}(v^P)\right), \\ \left(\tilde{\lambda}^P_{ID_1} \circ \tilde{\lambda}^P_{ID_2}\right)\left(({u_1}^P, v^P)({u_2}^P, v^P)\right) = \max\left(\tilde{\lambda}^P_{ID_1}(u_1^P u_2^P), \tilde{\lambda}^P_{IB_2}(v^P)\right) \\ \left(\tilde{\lambda}^N_{IC_1} \circ \tilde{\lambda}^N_{IC_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = r^N \max\left(\tilde{\lambda}^N_{IC_1}(u_1^N u_2^N), \tilde{\lambda}^N_{IA_2}(v^N)\right), \\ \left(\tilde{\lambda}^N_{ID_1} \circ \tilde{\lambda}^N_{ID_2}\right)\left(({u_1}^N, v^N)({u_2}^N, v^N)\right) = \min\left(\tilde{\lambda}^N_{ID_1}(u_1^N u_2^N), \tilde{\lambda}^N_{IB_2}(v^N)\right) \end{cases}$$

$$\left\{\begin{array}{c} (\tilde\gamma^P_{FC_1}\circ\tilde\gamma^P_{FC_2})(({u_1}^P,v^P)({u_2}^P,v^P)) = r^P\max\left(\tilde\gamma^P_{FC_1}(u_1^Pu_2^P),\tilde\gamma^P_{FA_2}(v^P)\right), \\ (\tilde\gamma^P_{FD_1}\circ\tilde\gamma^P_{FD_2})(({u_1}^P,v^P)({u_2}^P,v^P)) = \min\left(\tilde\gamma^P_{FD_1}(u_1^Pu_2^P),\tilde\gamma^P_{FB_2}(v^P)\right) \\ (\tilde\gamma^N_{FC_1}\circ\tilde\gamma^N_{FC_2})(({u_1}^N,v^N)({u_2}^N,v^N)) = r^N\min\left(\tilde\gamma^N_{FC_1}(u_1^Nu_2^N),\tilde\gamma^N_{FA_2}(v^N)\right), \\ (\tilde\gamma^N_{FD_1}\circ\tilde\gamma^N_{FD_2})(({u_1}^N,v^N)({u_2}^N,v^N)) = \max\left(\tilde\gamma^N_{FD_1}(u_1^Nu_2^N),\tilde\gamma^N_{FB_2}(v^N)\right) \end{array}\right\}$$

(iv) $\forall\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right),\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right)\in E^\circ - E$

$$\left\{\begin{array}{c} (\tilde\mu^P_{TC_1}\circ\tilde\mu^P_{TC_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = r^P\min(\tilde\mu^P_{TA_2}({v_1}^P),\tilde\mu^P_{TA_2}({v_2}^P),\tilde\mu^P_{TC_1}(u_1^Pu_2^P)), \\ (\tilde\mu^P_{TD_1}\circ\tilde\mu^P_{TD_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = \max\left(\tilde\mu^P_{TB_2}({v_1}^P),\tilde\mu^P_{TB_2}({v_2}^P),\tilde\mu^P_{TD_1}(u_1^Pu_2^P)\right) \\ (\tilde\mu^N_{TC_1}\circ\tilde\mu^N_{TC_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = r^N\max\left(\tilde\mu^N_{TA_2}({v_1}^N),\tilde\mu^N_{TA_2}({v_2}^N),\tilde\mu^N_{TC_1}(u_1^Nu_2^N)\right), \\ (\tilde\mu^N_{TD_1}\circ\tilde\mu^N_{TD_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = \min\left(\tilde\mu^N_{TB_2}({v_1}^N),\tilde\mu^N_{TB_2}({v_2}^N),\tilde\mu^N_{TD_1}(u_1^Nu_2^N)\right) \end{array}\right\}$$

$$\left\{\begin{array}{c} (\tilde\lambda^P_{IC_1}\circ\tilde\lambda^P_{IC_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = r^P\min\left(\tilde\lambda^P_{IA_2}({v_1}^P),\tilde\lambda^P_{IA_2}({v_2}^P),\tilde\lambda^P_{IC_1}(u_1^Pu_2^P)\right), \\ (\tilde\lambda^P_{ID_1}\circ\tilde\lambda^P_{ID_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = \max\left(\tilde\lambda^P_{IB_2}({v_1}^P),\tilde\lambda^P_{IB_2}({v_2}^P),\tilde\lambda^P_{ID_1}(u_1^Pu_2^P)\right) \\ (\tilde\lambda^N_{IC_1}\circ\tilde\lambda^N_{IC_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = r^N\max\left(\tilde\lambda^N_{IA_2}({v_1}^N),\tilde\lambda^N_{IA_2}({v_2}^N),\tilde\lambda^N_{IC_1}(u_1^Nu_2^N)\right), \\ (\tilde\lambda^N_{ID_1}\circ\tilde\lambda^N_{ID_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = \min\left(\tilde\lambda^N_{IB_2}({v_1}^N),\tilde\lambda^N_{IB_2}({v_2}^N),\tilde\lambda^N_{IB_1}(u_1^Nu_2^N)\right) \end{array}\right\}$$

$$\left\{\begin{array}{c} (\tilde\gamma^P_{FC_1}\circ\tilde\gamma^P_{FC_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = r^P\max\left(\tilde\gamma^P_{FA_2}({v_1}^P),\tilde\gamma^P_{FA_2}({v_2}^P),\tilde\gamma^P_{FC_1}(u_1^Pu_2^P)\right), \\ (\tilde\gamma^P_{FD_1}\circ\tilde\gamma^P_{FD_2})\left((u_1^P,v_1^P)(u_2^P,v_2^P)\right) = \min\left(\tilde\gamma^P_{FB_2}({v_1}^P),\tilde\gamma^P_{FB_2}({v_2}^P),\tilde\gamma^P_{FD_1}(u_1^Pu_2^P)\right) \\ (\tilde\gamma^N_{FC_1}\circ\tilde\gamma^N_{FC_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = r^N\min\left(\tilde\gamma^N_{FA_2}({v_1}^N),\tilde\gamma^N_{FA_2}({v_2}^N),\tilde\gamma^N_{FC_1}(u_1^Nu_2^N)\right), \\ (\tilde\gamma^N_{FD_1}\circ\tilde\gamma^N_{FD_2})\left((u_1^N,v_1^N)(u_2^N,v_2^N)\right) = \max\left(\tilde\gamma^N_{FB_2}({v_1}^N),\tilde\gamma^N_{FB_2}({v_2}^N),\tilde\gamma^N_{FD_1}(u_1^Nu_2^N)\right) \end{array}\right\}$$

Example 3.4 Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two fuzzy graphs, where $V_1 = (u, v)$ and $V_2 = (x, y)$. Suppose M_1 and M_2 be the bipolar neutrosophic fuzzy cubic set representations of V_1 and V_2. Also N_1 and N_2 be the bipolar neutrosophic fuzzy cubic set representations of E_1 and E_2 and defined as

$${M_1}^P = \left\langle \begin{array}{c} \{u,([0.4,0.5],0.1),([0.1,0.1],0.4),([0.7,0.8],0.2)\} \\ \{v,([0.3,0.4],0.2),([0.1,0.2],0.1),([0.4,0.5],0.5)\} \end{array} \right\rangle$$

$${M_1}^N = \left\langle \begin{array}{c} \{u,([-0.4,-0.5],-0.1),([-0.1,-0.1],-0.4),([-0.7,-0.8],-0.2)\} \\ \{v,([-0.3,-0.4],-0.2),([-0.1,-0.2],-0.1),([-0.4,-0.5],-0.5)\} \end{array} \right\rangle$$

$${N_1}^P = \langle\{uv,([0.3,0.4],0.2),([0.1,0.1],0.4),([0.7,0.8],0.2)\}\rangle$$

$${N_1}^N = \langle\{uv,([-0.3,-0.4],-0.2),([-0.1,-0.1],-0.4),([-0.7,-0.8],-0.2)\}\rangle$$

and

$$M_2{}^P = \left\langle \begin{matrix} \{x, ([0.5, 0.6], 0.3), ([0.7, 0.8], 0.7), ([0.1, 0.1], 0.5)\} \\ \{y, ([0.2, 0.3], 0.6), ([0.5, 0.6], 0.4), ([0.8, 0.9], 0.8)\} \end{matrix} \right\rangle$$

$$M_2{}^N = \left\langle \begin{matrix} \{x, ([-0.5, -0.6], -0.3), ([-0.7, -0.8], -0.7), ([-0.7, -0.8], -0.2)\} \\ \{y, ([-0.2, -0.3], -0.6), ([-0.5, -0.6], -0.4), ([-0.8, -0.9], -0.8)\} \end{matrix} \right\rangle$$

$$N_2{}^P = \langle \{xy, ([0.2, 0.3], 0.6), ([0.5, 0.6], 0.7), ([0.8, 0.9], 0.5)\} \rangle$$

$$N_2{}^N = \langle \{xy, ([-0.2, -0.3], -0.6), ([-0.5, -0.6], -0.7), ([-0.8, -0.9], -0.5)\} \rangle$$

Clearly $G_1 = \left((M_1{}^P, N_1{}^P), (M_1{}^N, N_1{}^N) \right)$ and $G_2 = \left((M_2{}^P, N_2{}^P), (M_2{}^N, N_2{}^N) \right)$ are bipolar neutrosophic cubic fuzzy graphs. So, the composition of two bipolar neutrosophic cubic fuzzy graphs G_1 and G_2 is again a bipolar neutrosophic cubic fuzzy graph, where

$$M_1^P\,[M_2^P] = \left\langle \begin{matrix} \{(u, x), ([0.4, 0.5], 0.3), ([0.1, 0.1], 0.7), ([0.7, 0.8], 0.2)\} \\ \{(u, y), ([0.2, 0.3], 0.6), ([0.1, 0.1], 0.4), ([0.8, 0.9], 0.2)\} \\ \{(v, x), ([0.3, 0.4], 0.3), ([0.1, 0.2], 0.7), ([0.4, 0.5], 0.5)\} \\ \{(v, y), ([0.2, 0.3], 0.6), ([0.1, 0.2], 0.4), ([0.8, 0.9], 0.5)\} \end{matrix} \right\rangle$$

$$M_1^N\,[M_2^N] = \left\langle \begin{matrix} \{(u, x), ([-0.4, -0.5], -0.3), ([-0.1, -0.1], -0.7), ([-0.7, -0.8], -0.2)\} \\ \{(u, y), ([-0.2, -0.3], -0.6), ([-0.1, -0.1], -0.4), ([-0.8, -0.9], -0.2)\} \\ \{(v, x), ([-0.3, -0.4], -0.3), ([-0.1, -0.2], -0.7), ([-0.4, -0.5], -0.5)\} \\ \{(v, y), ([-0.2, -0.3], -0.6), ([-0.1, -0.2], -0.4), ([-0.8, -0.9], -0.5)\} \end{matrix} \right\rangle$$

$$N_1^P\,[N_2^P] = \left\langle \begin{matrix} \{((u, x), (u, y)), ([0.2, 0.3], 0.6), ([0.1, 0.1], 0.7), ([0.8, 0.9], 0.2)\} \\ \{((u, y), (v, y)), ([0.2, 0.3], 0.6), ([0.1, 0.1], 0.4), ([0.8, 0.9], 0.2)\} \\ \{((v, y), (v, x)), ([0.2, 0.3], 0.6), ([0.1, 0.2], 0.7), ([0.8, 0.9], 0.5)\} \\ \{((v, x), (u, x)), ([0.3, 0.4], 0.3), ([0.1, 0.1], 0.7), ([0.7, 0.8], 0.2)\} \\ \{((u, x), (v, y)), ([0.2, 0.3], 0.6), ([0.1, 0.1], 0.7), ([0.8, 0.9], 0.2)\} \\ \{((u, y), (v, x)), ([0.2, 0.3], 0.6), ([0.1, 0.1], 0.7), ([0.8, 0.9], 0.2)\} \end{matrix} \right\rangle$$

$$N_1^N\ [N_2^N] = \left\langle \begin{matrix} \{((u,x),(u,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \\ \{((u,y),(v,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.4),([-0.8,-0.9],-0.2)\} \\ \{((v,y),(v,x)),([-0.2,-0.3],-0.6),([-0.1,-0.2],-0.7),([-0.8,-0.9],-0.5)\} \\ \{((v,x),(u,x)),([-0.3,-0.4],-0.3),([-0.1,-0.1],-0.7),([-0.7,-0.8],-0.2)\} \\ \{((u,x),(v,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \\ \{((u,y),(v,x)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \end{matrix} \right\rangle$$

Proposition 3.5 Let $G_1 = \left(({M_1}^P,{N_1}^P),({M_1}^N,{N_1}^N)\right)$ and $G_2 = \left(({M_2}^P,{N_2}^P),({M_2}^N,{N_2}^N)\right)$ be two bipolar neutrosophic cubic fuzzy graphs, then the Cartesian product of two bipolar neutrosophic cubic fuzzy graphs is again a bipolar neutrosophic cubic fuzzy graph.

Proof: Condition is oblivious for $({M_1}^P,{M_1}^N)\times({M_2}^P,{M_2}^N)$. Therefore we verify conditions only for $({N_1}^P,{N_1}^N)\times({N_2}^P,{N_2}^N)$, where

$$({N_1}^P,{N_1}^N)\times({N_2}^P,{N_2}^N) = \left\{ \begin{matrix} \left((\tilde{\mu}^P_{TC_1\times TC_2},\tilde{\mu}^P_{TD_1\times TD_2}),(\tilde{\mu}^N_{TC_1\times TC_2},\tilde{\mu}^N_{TD_1\times TD_2})\right),\left((\tilde{\lambda}^P_{IC_1\times IC_2,ID_1\times ID_2}),(\tilde{\lambda}^P_{IC_1\times IC_2,ID_1\times ID_2})\right), \\ \left((\tilde{\gamma}^P_{FC_1\times FC_2,FD_1\times FD_2}),(\tilde{\gamma}^N_{FC_1\times FC_2,FD_1\times FD_2})\right) \end{matrix} \right\}$$

Let $(u^P,u^N)\in V_1$ and $u_2v_2\in E_2$.

Then

$$\left(\tilde{\mu}^P_{TC_1\times TC_2}\left((u^P,u_2^P)(u^P,v_2^P)\right),\tilde{\mu}^N_{TC_1\times TC_2}\left((u^N,u_2^N)(u^N,v_2^N)\right)\right) = \left\{r^P\min\left\{\left(\tilde{\mu}^P_{TA_1}(u^P),\tilde{\mu}^P_{TC_2}({u_2}^P,v_2^P)\right)\right\},r^N\max\left\{\left(\tilde{\mu}^N_{TA_1}(u^P),\tilde{\mu}^N_{TC_2}({u_2}^P,v_2^P)\right)\right\}\right\}$$

$$\le \left\{ \begin{matrix} r^P\min\left\{\left(\tilde{\mu}^P_{TA_1}(u^P),\quad r^P\min\left(\tilde{\mu}^P_{TA_2}({u_2}^P),\tilde{\mu}^P_{TA_2}({v_2}^P)\right)\right)\right\}, \\ r^N\max\left\{\left(\tilde{\mu}^N_{TA_1}(u^N),\quad r^N\max\left(\tilde{\mu}^N_{TA_2}({u_2}^N),\tilde{\mu}^N_{TA_2}({v_2}^N)\right)\right)\right\} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} r^P\min\left\{r^P\min\left(\left(\tilde{\mu}^P_{TA_1}(u^P),\tilde{\mu}^P_{TA_2}({u_2}^P)\right),r^P\min\left(\tilde{\mu}^P_{TA_1}(u^P),\tilde{\mu}^P_{TA_2}({v_2}^P)\right)\right)\right\}, \\ r^N\max\left\{r^N\max\left(\left(\tilde{\mu}^N_{TA_1}(u^N),\tilde{\mu}^N_{TA_2}({u_2}^N)\right),r^N\max\left(\tilde{\mu}^N_{TA_1}(u^N),\tilde{\mu}^N_{TA_2}({v_2}^N)\right)\right)\right\} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} r^P\min\{(\tilde{\mu}^P_{TA_1}\times\tilde{\mu}^P_{TA_2})(u^P,u_2^P),(\tilde{\mu}^P_{TA_1}\times\tilde{\mu}^P_{TA_2})(u^P,v_2^P)\}, \\ r^N\max\{(\tilde{\mu}^N_{TA_1}\times\tilde{\mu}^N_{TA_2})(u^N,u_2^N),(\tilde{\mu}^N_{TA_1}\times\tilde{\mu}^N_{TA_2})(u^N,v_2^N)\} \end{matrix} \right\}$$

$$\left(\tilde{\mu}^P_{TD_1\times TD_2}\left((u^P,u_2^P)(u^P,v_2^P)\right),\tilde{\mu}^N_{TD_1\times TD_2}\left((u^N,u_2^N)(u^N,v_2^N)\right)\right) = \{\max\left\{\left(\tilde{\mu}^P_{TB_1}(u^P),\tilde{\mu}^P_{TD_2}({u_2}^P,v_2^P)\right)\right\},\min\left\{\left(\tilde{\mu}^N_{TB_1}(u^N),\tilde{\mu}^N_{TD_2}({u_2}^N,v_2^N)\right)\right\}$$

$$\leq \begin{Bmatrix} \max\{(\tilde{\mu}^P_{TB_1}(u^P), max(\tilde{\mu}^P_{TB_2}(u_2{}^P)), \tilde{\mu}^P_{TB_2}(v_2{}^P)\}, \\ \min\{(\tilde{\mu}^N_{TB_1}(u^N), min(\tilde{\mu}^N_{TB_2}(u_2{}^N)), \tilde{\mu}^N_{TB_2}(v_2{}^N)\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \max\left\{\max\left(\tilde{\mu}^P_{TB_1}(u^P), \tilde{\mu}^P_{TB_2}(u_2{}^P)\right), \max\left(\tilde{\mu}^P_{TB_1}(u^P), \tilde{\mu}^P_{TB_2}(v_2^P)\right)\right\} \\ \min\left\{\min\left(\tilde{\mu}^N_{TB_1}(u^N), \tilde{\mu}^N_{TB_2}(u_2{}^N)\right), \min\left(\tilde{\mu}^N_{TB_1}(u^N), \tilde{\mu}^N_{TB_2}(v_2{}^N)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \max\left\{(\tilde{\mu}^P_{TB_1} \times \tilde{\mu}^P_{TB_2})(u^P, u_2^P), \left((\tilde{\mu}^P_{TB_1} \times \tilde{\mu}^P_{TB_2})(u^P, v_2^P)\right)\right\}, \\ \min\left\{(\tilde{\mu}^N_{TB_1} \times \tilde{\mu}^N_{TB_2})(u^N, u_2^N), \left((\tilde{\mu}^N_{TB_1} \times \tilde{\mu}^N_{TB_2})(u^N, v_2^N)\right)\right\} \end{Bmatrix}$$

$$\left(\tilde{\lambda}^P_{IC_1 \times IC_2}\left((u^P, u_2^P)(u^P, v_2^P)\right), \tilde{\lambda}^N_{IC_1 \times IC_2}\left((u^N, u_2^N)(u^N, v_2^N)\right)\right)$$
$$= \left\{r^P \min\left\{\left(\tilde{\lambda}^P_{IA_1}(u^P), \tilde{\lambda}^P_{IC_2}(u_2{}^P, v_2^P)\right)\right\},\ \left\{\left(\tilde{\lambda}^N_{IA_1}(u^N), \tilde{\lambda}^N_{IC_2}(u_2{}^N, v_2^N)\right)\right\}\right\}$$

$$\leq \begin{Bmatrix} r^P \min\left\{\left(\tilde{\lambda}^P_{IA_1}(u^P), \quad r^P \min\left(\tilde{\lambda}^P_{IA_2}(u_2{}^P), \tilde{\lambda}^P_{IA_2}(v_2{}^P)\right)\right)\right\}, \\ r^N \max\left\{\left(\tilde{\lambda}^N_{IA_1}(u^N), \quad r^N \max\left(\tilde{\lambda}^N_{IA_2}(u_2{}^N), \tilde{\lambda}^N_{IA_2}(v_2{}^N)\right)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} r^P \min\left\{r^P \min\left(\left(\tilde{\lambda}^P_{IA_1}(u^P), \tilde{\lambda}^P_{IA_2}(u_2{}^P)\right), r^P \min\left(\tilde{\lambda}^P_{IA_1}(u^P), \tilde{\lambda}^P_{IA_2}(v_2{}^P)\right)\right)\right\}, \\ r^N \min\left\{r^N \max\left(\left(\tilde{\lambda}^N_{IA_1}(u^N), \tilde{\lambda}^N_{IA_2}(u_2{}^N)\right), r^N \min\left(\tilde{\lambda}^N_{IA_1}(u^N), \tilde{\lambda}^N_{IA_2}(v_2{}^N)\right)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} r^P \min\{(\tilde{\lambda}^P_{IA_1} \times \tilde{\lambda}^P_{IA_2})(u^P, u_2^P), (\tilde{\lambda}^P_{IA_1} \times \tilde{\lambda}^P_{IA_2})(u^P, v_2^P)\}, \\ r^N \max\{(\tilde{\lambda}^N_{IA_1} \times \tilde{\lambda}^N_{IA_2})(u^N, u_2^N), (\tilde{\lambda}^N_{IA_1} \times \tilde{\lambda}^N_{IA_2})(u^N, v_2^N)\} \end{Bmatrix}$$

$$\left(\tilde{\lambda}^P_{ID_1 \times ID_2}\left((u^P, u_2^P)(u^P, v_2^P)\right), \tilde{\lambda}^N_{ID_1 \times ID_2}\left((u^N, u_2^N)(u^N, v_2^N)\right)\right)$$
$$= \{\max\left\{\left(\tilde{\lambda}^P_{IB_1}(u^P), \tilde{\lambda}^P_{ID_2}(u_2{}^P, v_2^P)\right)\right\}, \min\left\{\left(\tilde{\lambda}^N_{IB_1}(u^N), \tilde{\lambda}^N_{ID_2}(u_2{}^N, v_2^N)\right)\right\}$$

$$\leq \begin{Bmatrix} \max\{(\tilde{\lambda}^P_{IB_1}(u^P), max(\tilde{\lambda}^P_{IB_2}(u_2{}^P)), \tilde{\lambda}^P_{IB_2}(v_2{}^P)\}, \\ \min\{(\tilde{\lambda}^N_{IB_1}(u^N), min(\tilde{\lambda}^N_{IB_2}(u_2{}^N)), \tilde{\lambda}^N_{IB_2}(v_2{}^N)\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \max\left\{\max\left(\tilde{\lambda}^P_{IB_1}(u^P), \tilde{\lambda}^P_{IB_2}(u_2{}^P)\right), \max\left(\tilde{\lambda}^P_{IB_1}(u^P), \tilde{\lambda}^P_{IB_2}(v_2^P)\right)\right\} \\ \min\left\{\min\left(\tilde{\lambda}^N_{IB_1}(u^N), \tilde{\lambda}^N_{IB_2}(u_2{}^N)\right), \min\left(\tilde{\lambda}^N_{IB_1}(u^N), \tilde{\lambda}^N_{IB_2}(v_2{}^N)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \max\left\{(\tilde{\lambda}^P_{IB_1} \times \tilde{\lambda}^P_{IB_2})(u^P, u_2^P), \left((\tilde{\lambda}^P_{IB_1} \times \tilde{\lambda}^P_{IB_2})(u^P, v_2^P)\right)\right\}, \\ \min\left\{(\tilde{\lambda}^N_{IB_1} \times \tilde{\lambda}^N_{IB_2})(u^N, u_2^N), \left((\tilde{\lambda}^N_{IB_1} \times \tilde{\lambda}^N_{IB_2})(u^N, v_2^N)\right)\right\} \end{Bmatrix}$$

$$\left(\tilde{\gamma}^P_{FC_1 \times FC_2}\left((u^P, u_2^P)(u^P, v_2^P)\right), \tilde{\gamma}^N_{FC_1 \times FC_2}\left((u^N, u_2^N)(u^N, v_2^N)\right)\right)$$
$$= \left\{r^P \max\left\{\left(\tilde{\gamma}^P_{FA_1}(u^P), \tilde{\gamma}^P_{FC_2}(u_2{}^P, v_2^P)\right)\right\}, \mathrm{r^N} \min\left\{\left(\tilde{\gamma}^N_{FA_1}(u^N), \tilde{\gamma}^N_{FC_2}(u_2{}^N, v_2^N)\right)\right\}\right\}$$

$$\leq \begin{Bmatrix} r^P \max\left\{\left(\tilde{\gamma}^P_{FA_1}(u^P), \quad r^P \max\left(\tilde{\gamma}^P_{FA_2}(u_2{}^P), \tilde{\gamma}^P_{FA_2}(v_2{}^P)\right)\right)\right\}, \\ r^N \min\left\{\left(\tilde{\gamma}^N_{FA_1}(u^N), \quad r^N \max\left(\tilde{\gamma}^N_{FA_2}(u_2{}^N), \tilde{\gamma}^N_{FA_2}(v_2{}^N)\right)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} r^P \max\left\{r^P \max\left(\left(\tilde{\gamma}^P_{FA_1}(u^P), \tilde{\gamma}^P_{FA_2}(u_2{}^P)\right), r^P \max\left(\tilde{\gamma}^P_{FA_1}(u^P), \tilde{\gamma}^P_{FA_2}(v_2{}^P)\right)\right)\right\}, \\ r^N \min\left\{r^N \min\left(\left(\tilde{\gamma}^N_{FA_1}(u^N), \tilde{\gamma}^N_{FA_2}(u_2{}^N)\right), r^N \min\left(\tilde{\gamma}^N_{FA_1}(u^N), \tilde{\gamma}^N_{FA_2}(v_2{}^N)\right)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} r^P \max\{(\tilde{\gamma}^P_{FA_1} \times \tilde{\gamma}^P_{FA_2})(u^P, u_2^P), (\tilde{\gamma}^P_{FA_1} \times \tilde{\gamma}^P_{FA_2})(u^P, v_2^P)\}, \\ r^N \min\{(\tilde{\gamma}^N_{FA_1} \times \tilde{\gamma}^N_{FA_2})(u^N, u_2^N), (\tilde{\gamma}^N_{FA_1} \times \tilde{\gamma}^N_{FA_2})(u^N, v_2^N)\} \end{Bmatrix}$$

$$\left(\tilde{\gamma}^P_{FD_1 \times FD_2}\left((u^P, u_2^P)(u^P, v_2^P)\right), \tilde{\gamma}^N_{FD_1 \times FD_2}\left((u^N, u_2^N)(u^N, v_2^N)\right)\right)$$
$$= \{\min\left\{\left(\tilde{\gamma}^P_{FB_1}(u^P), \tilde{\gamma}^P_{FD_2}(u_2{}^P, v_2^P)\right)\right\}, \max\left\{\left(\tilde{\gamma}^N_{FB_1}(u^N), \tilde{\gamma}^N_{FD_2}(u_2{}^N, v_2^N)\right)\right\}$$

$$\leq \begin{Bmatrix} \min\left\{\min\left(\tilde{\gamma}^P_{FB_1}(u^P), \tilde{\gamma}^P_{FB_2}(u_2{}^P)\right), \min\left(\tilde{\gamma}^P_{FB_1}(u^P), \tilde{\gamma}^P_{FB_2}(v_2{}^P)\right)\right\}, \\ \max\left\{\max\left(\tilde{\gamma}^N_{FB_1}(u^N), \tilde{\gamma}^N_{FB_2}(u_2{}^N)\right), \max\left(\tilde{\gamma}^N_{FB_1}(u^N), \tilde{\gamma}^N_{FB_2}(u_2{}^N)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \min\left\{\min\left(\tilde{\gamma}^P_{FB_1}(u^P), \tilde{\gamma}^P_{FB_2}(u_2{}^P)\right), \min\left(\tilde{\gamma}^P_{FB_1}(u^P), \tilde{\gamma}^P_{FB_2}(v_2^P)\right)\right\} \\ \max\left\{\max\left(\tilde{\gamma}^N_{FB_1}(u^N), \tilde{\gamma}^N_{FB_2}(u_2{}^N)\right), \max\left(\tilde{\gamma}^N_{FB_1}(u^N), \tilde{\gamma}^N_{FB_2}(v_2{}^N)\right)\right\} \end{Bmatrix}$$

$$= \begin{Bmatrix} \min\left\{(\tilde{\gamma}^P_{FB_1} \times \tilde{\gamma}^P_{FB_2})(u^P, u_2^P), \left((\tilde{\gamma}^P_{FB_1} \times \tilde{\gamma}^P_{FB_2})(u^P, v_2^P)\right)\right\}, \\ \max\left\{(\tilde{\gamma}^N_{FB_1} \times \tilde{\gamma}^N_{FB_2})(u^N, u_2^N), \left((\tilde{\gamma}^N_{FB_1} \times \tilde{\gamma}^N_{FB_2})(u^N, v_2^N)\right)\right\} \end{Bmatrix}$$

Similarly we can prove it for $w \in V_2$ and $u_1 v_1 \in E_1$.

Proposition 3.6 Let $G_1 = \left((M_1{}^P, N_1{}^P), (M_1{}^N, N_1{}^N)\right)$ and $G_2 = \left((M_2{}^P, N_2{}^P), (M_2{}^N, N_2{}^N)\right)$ be two bipolar neutrosophic cubic fuzzy graphs, then the composition of two bipolar neutrosophic cubic fuzzy graphs is again a bipolar neutrosophic cubic fuzzy graph.

Example 3.7 Let $G_1 = \left((M_1{}^P, N_1{}^P), (M_1{}^N, N_1{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = (V_1, E_1)$ where $v_1 = \{u, v, w\}, E = \{uv, vw, uw\}$

$$M_1{}^P = \left\langle \begin{matrix} \{u, ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.5, 0.6], 0.1)\} \\ \{v, ([0.1, 0.3], 0.1), ([0.4, 0.5], 0.3), ([0.1, 0.1], 0.2)\} \\ \{w, ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\} \end{matrix} \right\rangle$$

$$N_1{}^P = \left\langle \begin{matrix} \{uv, ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.5, 0.6], 0.1)\} \\ \{vw, ([0.1, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\} \\ \{uw, ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.5, 0.6], 0.1)\} \end{matrix} \right\rangle$$

$$\{u, ([-0.1, -0.1], -0.4), ([-0.3, -0.4], -0.2), ([-0.5, -0.6], -0.1)\}$$

$$M_1{}^N = \langle \{v, ([-0.1, -0.3], -0.1), ([-0.4, -0.5], -0.3), ([-0.1, -0.1], -0.2)\} \rangle$$

$$\{w, ([-0.2, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.3, -0.5], -0.2)\}$$

$$\{uv, ([-0.1, -0.1], -0.4), ([-0.3, -0.4], -0.2), ([-0.5, -0.6], -0.1)\}$$

$$N_1{}^N = \langle\{vw, ([-0.1, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.3, -0.5], -0.2)\}\rangle$$

$$\{uw, ([-0.2, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.5, -0.6], -0.1)\}$$

and $G_2 = \left((M_2{}^P, N_2{}^P), (M_2{}^N, N_2{}^N)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^* = (V_2, E_2)$ where $V_1 = \{a, v, c\}$ and $E_2 = \{ab, bc, ac\}$

$$\{a, ([0.6, 0.7], 0.5), ([0.1, 0.3], 0.4), ([0.2, 0.3], 0.6)\}$$

$$M_2{}^P = \langle\{b, ([0.1, 0.2], 0.3), ([0.5, 0.6], 0.2), ([0.8, 0.9], 0.4)\}\rangle$$

$$\{c, ([0.3, 0.4], 0.1), ([0.2, 0.3], 0.1), ([0.5, 0.6], 0.3)\}$$

$$\{ab, ([0.1, 0.2], 0.5), ([0.1, 0.3], 0.4), ([0.8, 0.9], 0.4)\}$$

$$N_2{}^P = \langle\{bc, ([0.1, 0.2], 0.3), ([0.2, 0.3], 0.2), ([0.8, 0.9], 0.3)\}\rangle$$

$$\{ac, ([0.3, 0.4], 0.5), ([0.1, 0.3], 0.4), ([0.5, 0.6], 0.3)\}$$

$$\{a, ([-0.6, -0.7], -0.5), ([-0.1, -0.3], -0.4), ([-0.2, -0.3], -0.6)\}$$

$$M_2{}^N = \langle \{b, ([-0.1, -0.2], -0.3), ([-0.5, -0.6], -0.2), ([-0.8, -0.9], -0.4)\} \rangle$$

$$\{c, ([-0.3, -0.4], -0.1), ([-0.2, -0.3], -0.1), ([-0.5, -0.6], -0.3)\}$$

$$\{ab, ([-0.1, -0.2], -0.5), ([-0.1, -0.3], -0.4), ([-0.8, -0.9], -0.4)\}$$

$$N_2{}^N = \langle \{bc, ([-0.1, -0.2], -0.3), ([-0.2, -0.3], -0.2), ([-0.8, -0.9], -0.3)\} \rangle$$

$$\{ac, ([-0.3, -0.4], -0.5), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.3)\}$$

then $G_1 \times G_2$ is a bipolar neutrosophic cubic fuzzy graph of $G_1^* \times G_2^*$,

where $V_1 \times V_2 = \{(u, a), (u, b), (u, c), (v, a), (v, b), (v, c), (w, a), (w, b), (w, c)\}$ and

$$M_1^P \times M_2^P = \left\langle \begin{array}{c} \{(u,a),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.5,0.6],0.1)\} \\ \{(u,b),([0.1,0.1],0.4),([0.3,0.4],0.2),([0.8,0.9],0.1)\} \\ \{(u,c),([0.1,0.1],0.6),([0.2,0.3],0.2),([0.5,0.6],0.1)\} \\ \{(v,a),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.2,0.3],0.2)\} \\ \{(v,b),([0.1,0.2],0.3),([0.4,0.5],0.3),([0.8,0.9],0.2)\} \\ \{(v,c),([0.1,0.3],0.1),([0.2,0.3],0.3),([0.5,0.6],0.2)\} \\ \{(w,a),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.3,0.5],0.2)\} \\ \{(w,b),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{(w,c),([0.2,0.3],0.1),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \end{array} \right\rangle$$

$$M_1^N \times M_2^N = \left\langle \begin{array}{c} \{(u,a),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.1)\} \\ \{(u,b),([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.8,-0.9],-0.1)\} \\ \{(u,c),([-0.1,-0.1],-0.6),([-0.2,-0.3],-0.2),([-0.5,-0.6],-0.1)\} \\ \{(v,a),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.2,-0.3],-0.2)\} \\ \{(v,b),([-0.1,-0.2],-0.3),([-0.4,-0.5],-0.3),([-0.8,-0.9],-0.2)\} \\ \{(v,c),([-0.1,-0.3],-0.1),([-0.2,-0.3],-0.3),([-0.5,-0.6],-0.2)\} \\ \{(w,a),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\} \\ \{(w,b),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{(w,c),([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \end{array} \right\rangle$$

$$N_1^P \times N_2^P = \left\langle \begin{matrix} \{((u,a),(u,b)),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.8,0.9],0.1)\} \\ \{((u,b),(u,c)),([0.1,0.1],0.4),([0.2,0.3],0.2),([0.8,0.9],0.1)\} \\ \{((u,a),(v,c)),([0.1,0.1],0.4),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,c)),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,b)),([0.1,0.2],0.5),([0.1,0.3],0.4),([0.8,0.9],0.2)\} \\ \{((v,b),(w,b)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,b),(w,c)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,a),(w,c)),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \\ \{((u,ab),(w,a)),([0.1,0.1],0.5),([0.1,0.2],0.6),([0.5,0.6],0.1)\} \end{matrix} \right\rangle$$

$$N_1^N \times N_2^N = \left\langle \begin{matrix} \{((u,a),(u,b)),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.1)\} \\ \{((u,b),(u,c)),([-0.1,-0.1],-0.4),([-0.2,-0.3],-0.2),([-0.8,-0.9],-0.1)\} \\ \{((u,a),(v,c)),([-0.1,-0.1],-0.4),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,c)),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,b)),([-0.1,-0.2],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.2)\} \\ \{((v,b),(w,b)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,b),(w,c)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,a),(w,c)),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \\ \{((u,ab),(w,a)),([-0.1,-0.1],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.1)\} \end{matrix} \right\rangle$$

Conclusion: In this paper ,introduced Cartesian product and composition of bipolar neutrosophic bipolar fuzzy graphs.we investigate some of their properties.

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