APPLICATION OF ECONOMIC AND MATHEMATICAL METHODS IN STUDYING INFLATION PROCESSES IN AZERBAIJAN: SARIMAX MODEL AND EXTENDED MODEL OF FOURIER SERIES

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In this study, inflation cycles and the forecasting quality of mathematical methods were applied and examined. SARIMAX models and an extended Fourier series model were applied in forecasting inflationary processes. The study identifies the cycles and intensity of inflation. The Fourier series model has been extended with a dummy variable (Extended Fourier Series Model) in order to consider economic shocks in the model.

In this study inflation cycle of Azerbaijan and forecasting quality of mathematics methods have been determined and investigated. Model SARIMAX and Extended Fourier series model have been implemented in forecasting the inflationary processes. Fourier series model has been extended by additive dummy variable (EFS model- Extended Fourier series model) in order to consider the economic shocks in the model.

Keywords: *Fourier, inflationary processes, forecasting, SARIMAX model*
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Introduction...

According to dialectics, development does not occur as a linear movement, but "spirally". There is also a return to the starting point here. But this return and periodic processes are taking place on new foundations. [1, p.272].

As periodic processes, inflationary processes occur from complex mutual economic events occurring in a dynamic economic system. The relevance of the doctrine of inflationary processes acquires a new quality based on the economic behavior of business entities, and this makes this study more interesting for researchers. In addition, in order to consider economic shocks in the economic model, the Fourier series were extended with a dummy variable.

Previous research in this area.

For the first time, the Fourier series were presented by the French mathematician Joseph Fourier in his treatises "Distribution of heat in solids" in 1807 and "Analytical theory of heat" in 1822 [4, p.125].

Andrea Fumi and others studied demand forecasts using Fourier series [2]. NS. Omekara, E. Ekpenyong, MP Ekerete, and also studied the dynamics of inflation in Nigeria using the Fourier series [3].

N. Liu, V. Babushkin and A. Afshari used this model in their work for short-term prediction of electrical loads. Maurice Omane and others have applied SARIMAX to forecast inflation in Ghana. Sani I. Doguva and Sara O. Alade used these models for short-term forecasting of inflation in Nigeria and others.

Despite the fact that a lot of research has been done in connection with inflation on various aspects in Azerbaijan, the analysis of Fourier series and SARIMAX models has not been implemented in the Azerbaijani economy to study inflation processes.

Unlike other studies, it accounts for the economic shock using a dummy variable.

Methodology and theoretical foundations of applied mathematical methods. The Fourier series is also applicable in econometrics, in addition to other areas such as electronics, optics, signal processing, acoustics, etc. [4; 12].

In general, harmonic oscillations change in accordance with the laws of sine and cosine. These changes will be brought to harmonic movements. The number of fluctuations hitting each month is the frequency of the fluctuations. The time it takes for a full swing is a period. Frequency and period are mutually reciprocal values. These are often periodic events, consisting of the summation of a series of harmonic vibrations. As a result, the oscillation graph forms harmonic oscillations [8]. To determine the inflation dynamics function, we briefly present a simple harmonic series:$\nu \ = \ 1 \ / \ TT \ = \ 1 \ / \ \nu$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx \ + b_n \sin nx \ )$ (2.1)

Series (2.1) is a trigonometric series of coefficients, which have the values, ,,,. In equal frequency harmonics, the price x is (). Series (2.1) is accumulated in this function for all values$a_0 a_1 \ a_2 \ ... \ b_1 \ b_2 \ ... \omega tx \ = \ \omega t^{\mathbf{X}}$... That is, the sum of the series is equal to the function.$f(x)$

Harmonic analysis of inflationary processes is an expansion of series in trigonometric series. In order to expand a periodic function with a period of 2π, it is necessary to assign the coefficients of this series. It can be calculated using the following formulas:$f(x) a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x) dx$; $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \ \cos nx \ dx$;

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \ dx$; $n = 1,2,3..$ (2.2)

Consequently, the series having the above coefficients is called the Fourier series with respect to the function $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx \ + b_n \sin nx \ ) \ f(x)$.

In this study, a variable is an argument dependent variable or function. The addition of simpler harmonics more accurately expresses the periodic motion defined by the function. Thus, Fourier series models are more tied to the length of the series. Therefore, the decomposition of the trigonometric series is of great importance [7]. Movements in aggregate demand and aggregate supply and market regulation seeking to ensure macroeconomic equilibrium form cyclical inflationary processes. And the definition of inflation cycles and the construction of models of the Fourier series are based on the Fourier coefficients and the Fourier frequency.$unц_t tf(x)$.

Fourier series analysis is more appropriate for identifying periodic signals of inflation. In this study, inflationary processes are investigated as the summation of simple harmonics. The given harmonics of inflationary processes are repeated every t = 2π time interval [5], in other words, they turn into periodic processes. The summation of the equal-frequency harmonics forms a simple harmonic.

To determine inflationary cycles, one needs to consider more complex harmonics with different frequencies of sine and cosine waves. Therefore, the model of the extended Fourier series with a dummy variable has the following form:

(2.3)$\widehat{unц}_t = \sum_{n=0}^{p} c_n t^n + \sum_{n=1}^{k}\left(\hat{a}_n \cos n\omega t \ + \hat{b}_n \sin n\omega t \ \right) + \Phi\Pi_t + u_t$

Here, the inflation rate regression estimates$\widehat{unц}_t$ –

$\sum_{n=0}^{p} c_n t^n -$ equation of estimated trends, parameter estimates, - {0,1} - fictitious exogenous variable, residuals or residual of the model$\hat{a}_n$ u $\hat{b}_n$,$(n =$ *1,2,...,q*$-\Phi\Pi tut-$

The coefficients are assessed as follows:a_n u b_n

$$\left.\begin{aligned} a_n &\approx \frac{2}{N}\sum_{t=1}^{N} \Delta uny_t \cos\omega_n t \\ b_n &\approx \frac{2}{N}\sum_{t=1}^{N} \Delta uny_t \sin\omega_n t \end{aligned}\right\} (2.4)$$

The intensity function for the frequency along the row is determined as follows [3]: $I(f_n) f_n$ипц$_t$

(2.5)$I(f_n) = \frac{N}{2}[a_n^2 + b_n^2], n = 1,2, \dots, q$

Since inflationary processes are cyclical, the - th harmonic of the frequency should be assigned to the interval. The Fourier frequency is used to determine the parameters of the Fourier models [3]. The greatest amount of intensity is determined by the cycles of inflationary processes. The trend equations can be specified by the following parameters:$f_n - n0 \leq f_n \leq 0.5$

(2.6)$uny_t = \sum_{n=0}^{p} c_n t^n$

Subtractions of the trend from the original series are calculated as follows: ипц

$\triangle \widehat{uny}_t = \sum_{n=1}^{k}\left(\hat{a}_n\, cos\, n\omega t\ + \hat{b}_n sin\, n\omega t\ \right)$ (2.7)

The structural model of the SARIMAX process is structurally different from the SARIMA model [10; 11; 13; 14; 15].

$\emptyset(L)\varphi(L^S)(1-L)^d(1-L^S)^D uny_t = \theta(L)\Theta(L^S)\epsilon_t$ (2.8)

Since the SARIMAX model includes exogenous variables. [eleven]. Consequently, the SARIMAX model was built on the consumer price index as a process:$\{ИПЦ_t\} t \in Z(p,d,q)\ (P,D,Q)[S]$

$\emptyset(L)\varphi(L^S)(1-L)^d(1-L^S)^D\left(uny_t - \psi^{`} X_t\right) = \theta(L)\Theta(L^S)\epsilon_t$ (2.9)

$$uny_t = \sum_{i=0}^{k} \alpha_i X_i + \frac{\theta(L)\Theta(L^S)}{\emptyset(L)\varphi(L^{S)}(1-L)^d(1-L^S)^D}\varepsilon_t$$

X_t $-$vectors of an exogenous variable, parameter estimate. the number of exogenous and integrated variables will be additive as follows:$\psi^{`} - k\{X_{it}, i = 1,2, \dots, k\}$

$uny_t = c + \sum_i^k \gamma_i X_{it} + \mu_t$

Here, autoregional members.$\{\mu_t\} t \in Z$

Statistical data.

In this study, statistics cover the period from January 1996 to March 2015. Statistical data are divided into two parts: calibration and control [2].

The information was acquired from the official statistics of the Central Bank of the Republic of Azerbaijan5...

Time series stationarity is important in econometrics [5].

Application of extended models of the Fourier series in the study of the cyclical analysis of inflationary processes.

The frequency cannot be seen due to the uncertainty of the schedule. Therefore, the highest intensity is determined based on the periodogram analysis. It was determined that the estimated frequency at the corresponding highest intensity is equal to. This is seen as the frequency of the wave of inflation. (See Table 1, Fig. 1),$f = 0.0826$

Table 1. Cyclic analysis and diagnostics inflationary processes

n	Intensity	The inflation cycle	Frequency	Amplitude $\boldsymbol{n}$-x fluctuations
1	17.55	218	0.0046	0.40
2	6.28	109	0.0092	0.24
...	...	...	...	...
eighteen	63.26	12.1	0.0826	0.76
...	...	...	...	...
109	0.14	2	0.5000	0.04

Source: Author's calculations and Microsoft Excel 2007.

To determine inflationary cycles, long-term periods were studied, which cover the impact of all economic shocks that occurred during 1996-2015. In the long term, cycle inflation rates have been determined over a 12-month period. Since, the rate of regulation of inflationary processes directed towards equilibrium is equal to 6 (six) months. Hence, markets bring inflation back to equilibrium when prices rise.$t = 2\pi$

[5] CBA official website, http://www.cbar.az

As a result of the analysis of the Fourier series, the inflation cycle was determined for 12 months (See Fig. 1, Table 1).

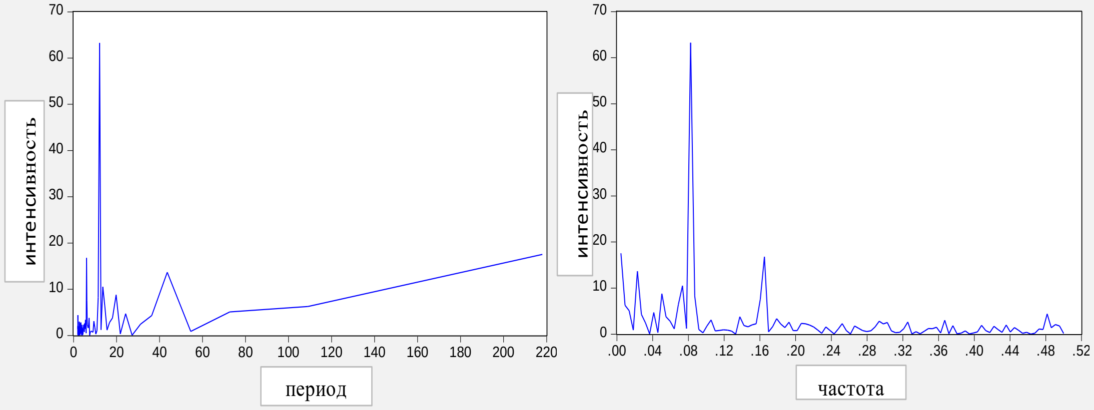
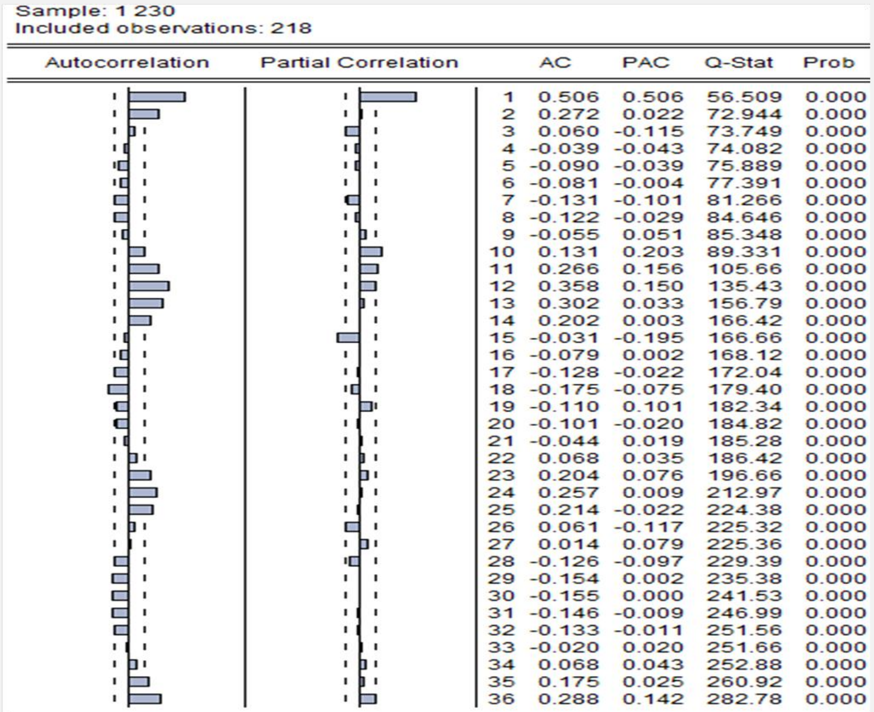


Fig. 1. Intensity and frequency of periods
Source: Author's calculations and Eviews 7

Below indicated correlogram analysis of ACF and PACF shows periodic effects (see Fig. 2).



Sample: 1 230
Included observations: 218

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.506	0.506	56.509	0.000
		2	0.272	0.022	72.944	0.000
		3	0.060	-0.115	73.749	0.000
		4	-0.039	-0.043	74.082	0.000
		5	-0.090	-0.039	75.889	0.000
		6	-0.081	-0.004	77.391	0.000
		7	-0.131	-0.101	81.266	0.000
		8	-0.122	-0.029	84.646	0.000
		9	-0.055	0.051	85.348	0.000
		10	0.131	0.203	89.331	0.000
		11	0.266	0.156	105.66	0.000
		12	0.358	0.150	135.43	0.000
		13	0.302	0.033	156.79	0.000
		14	0.202	0.003	166.42	0.000
		15	-0.031	-0.195	166.66	0.000
		16	-0.079	0.002	168.12	0.000
		17	-0.128	-0.022	172.04	0.000
		18	-0.175	-0.075	179.40	0.000
		19	-0.110	0.101	182.34	0.000
		20	-0.101	-0.020	184.82	0.000
		21	-0.044	0.019	185.28	0.000
		22	0.068	0.035	186.42	0.000
		23	0.204	0.076	196.66	0.000
		24	0.257	0.009	212.97	0.000
		25	0.214	-0.022	224.38	0.000
		26	0.061	-0.117	225.32	0.000
		27	0.014	0.079	225.36	0.000
		28	-0.126	-0.097	229.39	0.000
		29	-0.154	0.002	235.38	0.000
		30	-0.155	0.000	241.53	0.000
		31	-0.146	-0.009	246.99	0.000
		32	-0.133	-0.011	251.56	0.000
		33	-0.020	0.020	251.66	0.000
		34	0.068	0.043	252.88	0.000
		35	0.175	0.025	260.92	0.000
		36	0.288	0.142	282.78	0.000

Fig. 2. ACF and PACF, correlogram

Source: Eviews 7

Thus, models with seasonal components can be constructed as follows:

$$\Delta uny_t = 0.7256\cos(\omega t) - 0.3137\sin(\omega t) - 0.3922\cos(2\omega t) + 0.2153\cos(12\omega t) + 0.3372\cos(13\omega t) + 0.1991\sin(37\omega t)$$

High frequency and harmonic fluctuations for inflationary processes are respectively equal to 0.0826 and 54.

The econometric estimates of the component show that the autonomous part is statistically significant (see Table 2).

Table 2.

Variable	Coefficients	Std.otk	t-statistics	probability
Trend component (c)	100.3359	0.156755	640.0793	0.0000
Trend component (@trend)	0.001112	0.00125	0.889897	0.3745
Residual component u (-1)	0.333744	0.06	5.185301	0.0000

4363

Source: Author's calculations and Eviews 7

Econometric models with a combination of Fourier series model components take the following form:

$$\text{ипц}_t = 100.4614 + 0.723093\cos(\omega t) - 0.289607\sin(\omega t) - 0.314703\cos(2\omega t) + 0.241366\cos(12\omega t) + 0.327632\cos(13\omega t) + 0.212861\sin(37\omega t) + 0.337403u_{t-1}$$

To account for economic shocks, the Fourier series models were extended with the FP dummy {0,1} as additive. Then the expansion of the Fourier series will look as follows. Here FP is a dummy variable and is defined as 0 and 1.

$$\text{ИПЦ}_t = 100.4157 + 0.673452\cos(\omega t) - 0.31071\sin(\omega t) - 0.318\cos(2\omega t) + 0.260353\cos(12\omega t) + 0.317045\cos(13\omega t) + 0.140359\sin(37\omega t) + 0.244708u_{t-1} + 1.484984\ \text{ФП}$$

When comparing the quality of models (RMSE, MAE, MAPE, TIC, Durbin-Watson statis. Criterion, Akaike info crit. Fourier series (RF) and extended Fourier series models (RF), it can be seen that RFRF models are more significant. Fuller show that the model has stationarity at all levels of statistical significance.

Modeling inflationary processes based on SARIMAX models.

As stated above, SARIMAX models are structurally different from SARIMA models. Since SARIMAX models include exogenous variables.

When adding dummy variables, considering economic shocks as exogenous variables, the SARIMAX model looks like this: $(1-\emptyset_1 L)\left(1-\varphi_1 L^{12}\right)\mu_t = (1+\theta_1 L)\varepsilon_t$

$$(1-0.4224L)(1-0.2420L^{12})\mu_t = (1+0.1019L)\varepsilon_t$$

$$\text{ипц}_t = c + \Theta\text{ФП}_t + \mu_t$$

$$\text{ипц}_t = 100.3304 + 2.085\text{ФП}_t + \frac{(1+0.1019L)\varepsilon_t}{(1-0.4224L)(1-0.2420L^{12})}$$

$$\text{ипц}_t = 100.3304 + 2.085\Phi D_t + \frac{\varepsilon_t + 0.1019\varepsilon_{t-1}}{1-0.24\text{ипц}_{t-12} - 0.422\text{ипц}_{t-1} + 0.1022\text{ипц}_{t-13}}$$

The evaluation results of the above constructed SARIMAX models give estimates of the model parameters in the following table.

Table 4. Estimation of parameters of SARIMAX models and quality of forecasting models.

Options	SARIMAX	Standard dev.	SARIMAX (102)	Standard

	(100) (100) [12]		(100) [12]	dev.
C	100.3318	[0.143681]	100.3304	[0.14932]
FP	2.060154	[0.481008]	2.085979	[0.489423]
AR (1)	0.454368	[0.063536]	0.422471	[0.064776]
SAR (12)	0.243506	[0.06088]	0.242089	[0.064191]
MA (2)			0.101964	[0.000537]
Determination coefficient	0.426393		0.42843	
RMSE	0.316724		0.15172	
MAE	0.316724		0.15172	
MAPE	0.315524		0.15117	
Akaike info criteria	2.516702		2.52290	
Schwarz criteria	2.581542		2.60395	
Darbin-Watson p.	2.007207		1.94838	

Source: Author's calculations and Eviews 7

When considering the forecasting results of the SARIMAX (100) (100) [12] and SARIMAX (102) (100) [12] models, it can be seen that the SARIMAX (1 0 2) (1 0 0) [12] models have a closer price to the actual indicator (See Table 4). The quality indicators of SARIMAX (1 0 2) (1 0 0) [12] models are considered more acceptable than SARIMAX (1 0 0) (1 0 0) [12]. The SARIMAX model consists of exogenous variables added to the SARIMA model. It is the integration of regression models to SARIMA models. This model combines the advantages of both models. ARIMA methods provide for autocorrelation in residuals. The comparative forecasting schedule for the SARIMAX and RRF models is described below (see Fig. 3, Table 5):

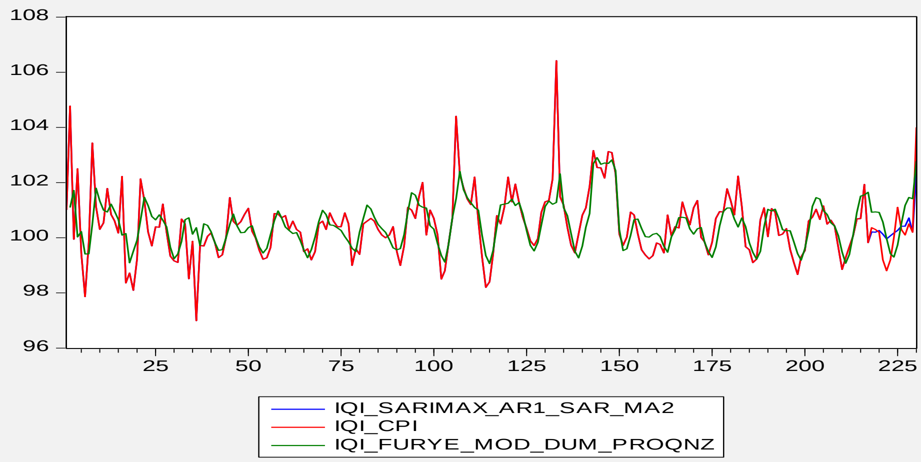


Fig. 3. SARIMAX Model Prediction and Extended Fourier Series
Source: Author's calculations and Eviews 7

Table 5. Estimation of SARIMAX model and Fourier series

n	SARIMAX (1 0 2) (1 0 0) [12] model	Rrf model	CPI
218	100.21	100.92	100.36
219	100.20	100.93	100.30
220	100.27	100.92	100.20
221	100.13	100.57	99.20
222	99.96	99.97	98.80
223	100.07	99.42	99.20
224	100.17	99.30	100.00
225	100.27	99.74	101.10
226	100.41	100.50	100.30
227	100.42	101.17	100.10
228	100.72	101.46	100.50
229	100.21	101.41	100.20
230	102.39	102.77	104.00

Consequently, all these models in forecasting only then acquire an approximation to the actual assessment, when all significant processes occurring in the economic system are formed on the basis of chance and retain their market mechanisms. These are the necessary conditions for obtaining the smallest deviations in predictive models. Evolutions in the behavior of economic subjects lead inflationary processes from one quality to another. In this case, a sufficient condition is to maintain a significant quality. These conditions increase the adequacy of the approximation of the models to the actual indicators.

Based on the results of the study, it was determined that the use of Fourier series and SARIMAX models is considered more significant in modeling and predicting inflationary processes in Azerbaijan. Consequently, the rate of regulation of inflationary processes aimed at equilibrium is equal to 6 (six) months. For all these models, forecasting only then acquires an approximation to the actual estimate, when all significant processes occurring in the economic system are formed on the basis of chance, and retain their market mechanisms. This is a necessary condition for the smallest deviations in predictive models. Evolutions in the behavior of economic entities transfer inflationary processes from one quality to another. In this case, a significant preservation of quality is a sufficient condition.

Consequently, the application of these methods in the study of inflationary processes is of theoretical and practical importance in determining the macroeconomic monetary policy.

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