Application of economic and mathematical methods in the study of inflation processes in Azerbaijan: Sarimax model and extended Fourier series model

Yusifov Murad Yusif oglu [1]

Abstract. In this study were applied and researched inflation cycles and forecasting quality mathematical methods . SARIMAX models and extended Fourier series model were applied in forecasting inflation processes. The study determines the cycles and intensity of inflation. The Fourier series model was extended with a dummy variable (AFS - extended Fourier series model) in order to consider economic shocks in the model.

Keywords: *Fourier series, inflation processes, forecasting, SARIMAX model*

Introducing

According to dialectic , development does not occur as a linear movement, but "spirally " . Here too there is a return to the initial moment. But this return and periodic processes occur on new foundations . [1, p.272].

As periodic processes , inflation processes occur from complex mutual economic events occurring in a dynamic economic system. The relevance of the teaching of inflation processes acquires a new quality on the basis of the economic behavior of economic entities, and this makes this study more interesting for researchers . In addition, in order to consider economic shocks in the model of the economy, the Fourier series was expanded with a dummy variable.

1. Previous research in this area.

For the first time , Fourier series was presented by the French mathematician Joseph Fourier in his treatises " Propagation of Heat in Solid Bodies " in 1807 and "Analytical Theory of Heat" in 1822 [4, p.125].

Andrea Fumi and others studied demand forecasts using Fourier series [2]. K. O. Omekara , E. Ekpenyong , M. P. Ekerete , and also studied the dynamics of inflation in Nigeria using Fourier series [3] .

[1] Baku State University, PhD student, E-mail: yusifovmy @ gmail . com

N. Liu, V. Babushkin and A. Afshari used this model in their work on short-term forecasting of electrical loads. Maurice Omane and others applied SARIMAX in forecasting inflation in Ghana. Sani I. Doguwa and Sarah O. Alade used these models in short-term forecasting of inflation in Nigeria, etc.

Although many studies have been done on inflation in various aspects in Azerbaijan, Fourier series analysis and SARIMAX models have not been implemented in the economy of Azerbaijan to study the inflation processes.

Unlike other studies, this study controls for economic shock using a dummy variable.

2. Methodology and theoretical foundations of applied mathematical methods

Fourier series is also applicable in econometrics , among other fields such as electronics , optics, signal processing, and mathematics, etc. [4 ;12].

In general, harmonic oscillations change in accordance with the laws of sine and cosine . These changes will be brought to harmonic movements. The number of oscillations, falling on each month , is the frequency oscillations. The time required for a complete oscillation, is the period of the ohm . Frequency $\nu = 1 / T$and period $T = 1 / \nu$are mutually inverse quantities. These are often periodic events consisting of the summation of a series of harmonic oscillations. As a result, the oscillation graph forms harmonic oscillations [8]. To determine the inflation dynamics function, we briefly present a simple harmonic series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \quad (2.1)$$

The series (2.1) is a trigonometric series of coefficients, which have values a_0, a_1, a_2 ..., b_1, b_2 .... In equal frequency harmonics a x the price x equal to ωt($x = \omega t$). The series (2.1) accumulates in this function for all values of x. That is, the sum of the series is equal to the function $f(x)$.

Harmonic analysis of inflationary processes is the expansion of series into trigonometric series . In order to expand a periodic function $f(x)$, having a period of 2π , it is necessary to assign coefficients to this series . This can be calculated using the following formulas:

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx ; \quad a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \, dx ;$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \, dx ; \quad n = 1,2,3.. \quad (2.2)$$

Therefore, the series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ having the above mentioned coefficients is called Fourier series of the $f(x)$.function

In this study , the variable $\text{ипц}_t$is a variable dependent on the argument $t$or a function . The addition of simpler harmonics more accurately expresses the periodic motion,defined by the function $f(x)$. Thus; the Fourier series models are more sensitive to the length of the series. Therefore, the decomposition of the trigonometric series is of great importance [7]. The movements occurring in aggregate demand and aggregate supply, and market regulation, striving to ensure macroeconomic equilibrium forms cyclical inflationary processes. And the definition of inflation cycles and the construction of Fourier series models are based on the Fourier coefficients and the Fourier frequency.

Fourier series analysis is more appropriate to determine periodic inflation signals. In this study, inflation processes are investigated as summations of simple harmonics. The given harmonics of inflation processes are repeated every t = 2π time interval [5], in other words, they turn into periodic processes. The summation of equal-frequency harmonics forms a simple harmonic.

To determine inflationary cycles, it is necessary to consider more complex harmonics, which have different frequencies of sine and cosine waves. Therefore, the model The extended Fourier series with a dummy variable has the following view :

$$\widehat{\text{ипц}}_t = \sum_{n=0}^{p} c_n t^n + \sum_{n=1}^{k}\left(\hat{a}_n\, cos\, n\omega t\ + \hat{b}_n sin\, n\omega t\ \right) + \Phi\Pi_t + u_t (2.3)$$

Here , $\widehat{\text{ипц}}_t$ –the inflation rate regression estimates are

$\sum_{n=0}^{p} c_n t^n$ –estimated trend equation ,$\hat{a}_n$ и $\hat{b}_n$, $(n = 1,2, \dots, q) -$ parameter estimates , $\Phi\Pi_t$- $\{0,1\}$ is a dummy exogenous variable, $u_t -$ remnants or residual of the model

Coefficients a_n и $b_n$are assessed as follows:

$$\left.\begin{aligned} a_n &\approx \frac{2}{N}\sum_{t=1}^{N} \Delta\text{ипц}_t \cos \omega_n t \\ b_n &\approx \frac{2}{N}\sum_{t=1}^{N} \Delta\text{ипц}_t \sin \omega_n t \end{aligned}\right\}(2 . 4)$$

Intensity function $I(f_n)$ for frequency $f_n$in a row ипц_t is defined as follows [3] :

$$I(f_n) = \frac{N}{2}[a_n^2 + b_n^2], n = 1,2, \dots, q (2.5)$$

Since inflationary processes are cyclical,$f_n - n$-th harmonic frequency must be assigned in the interval $0 \leq f_n \leq 0.5$. The Fourier frequency is used to determine the parameters of the model . Fourier [3]. The greatest amount of intensity is determined by the cycles of inflationary processes. The trend equations can be specified using the following parameters :

$$\text{ипц}_t = \sum_{n=0}^{p} c_n t^n \qquad (2 . 6)$$

Subtracting the trend from the original series ипцare calculated as follows:

$$\Delta\ \widehat{\text{ипц}}_t = \sum_{n=1}^{k}\left(\hat{a}_n\, cos\, n\omega t\ + \hat{b}_n sin\, n\omega t\ \right)(2 . 7)$$

structural model is structurally different from the SARIMA model [10;11;13;14;15] .

$$\emptyset(L)\varphi(L^S)(1-L)^d(1-L^S)^D\text{ипц}_t = \theta(L)\Theta(L^S)\epsilon_t (2.8)$$

Since the SARIMAX model includes exogenous variables . [11] Therefore, the SARIMAX model was constructed based on the consumer price index $\{ИПЦ_t\}t \in Z$as a (p,d,q) $(P,D,Q)[S]$process:

$$\emptyset(L)\varphi(L^S)(1-L)^d(1-L^S)^D\left(ипц_t - \psi^{`}X_t\right) = \theta(L)\Theta(L^S)\epsilon_t \qquad (2.9)$$

$$ипц_t = \sum_{i=0}^{k} \alpha_i X_i + \frac{\theta(L)\Theta(L^S)}{\emptyset(L)\varphi(L^{S)}(1-L)^d(1-L^S)^D}\varepsilon_t$$

X_t –vectors of exogenous variable , $\psi^{`}$ –Parameter estimation. $k$number of exogenous and integrated $\{X_{it}, i = 1,2,\dots,k\}$ variables will be additive as follows :

$ипц_t = c + \sum_i^k \gamma_i X_{it} + \mu_t$

Here,$\{\mu_t\}t \in Z$ autoregional members.

3. Statistical data .

In this study, the statistics cover the period from January 1996 to March 2015 . Statistical data are divided into two parts: calibration and control [2].

The information was obtained from the official statistical data of the Central Bank of the Republic of Azerbaijan[2].

Time series stationarity is of great importance in econometrics [5].

4. Application of extended Fourier series models in the study of cyclical analysis of inflationary processes .

The periodicity cannot be seen due to the uncertainty of the graph. Therefore, the highest intensity is determined based on the periodogram analysis. It was determined that the frequency estimate for the corresponding highest intensity is $f = 0.0826$. This is considered as the frequency of the inflation wave . (See Table 1 , Fig. 1),

Table 1. Cyclic analysis and diagnostics inflationary processes

n	Intensity	Inflation cycle	frequency	Amplitude a n- x oscillations
1	17.55	218	0.0046	0.40
2	6.28	109	0.0092	0.24
.	.	.	.	.
18	**63.26**	**12.1**	**0.0826**	**0.76**
.	.	.	.	.

[2] Official website of the Central Bank of the Russian Federation, http :// www . cbar . az

109	0.14	2	0.5000	0.04

Source: Author skiie calculations and Microsoft Excel 20 07 .

To determine inflation cycles, long-term periods were studied, which cover the impact of all economic shocks that occurred during 1996-2015. In the long term, the inflation cycle indicators were determined for a period of 12 months. Since $t = 2\pi$, the rate of regulation of inflation processes directed to the state of equilibrium is 6 (six) months. Consequently, markets return inflation to the state of equilibrium when the price increases.

As a result of the analysis of Fourier series, the inflation cycle was determined for 12 months (See Fig. 1, Table 1).

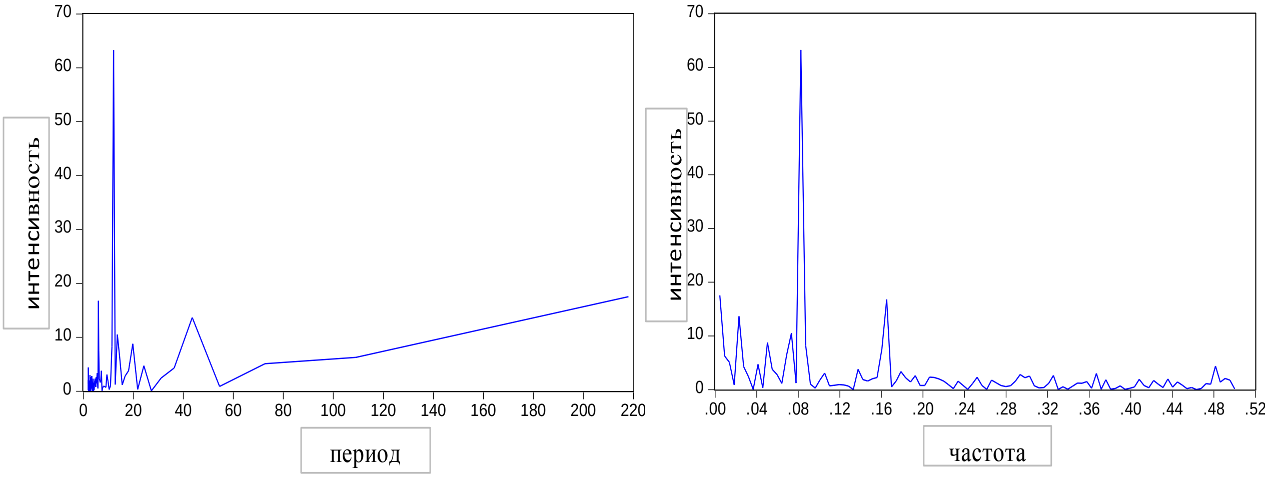
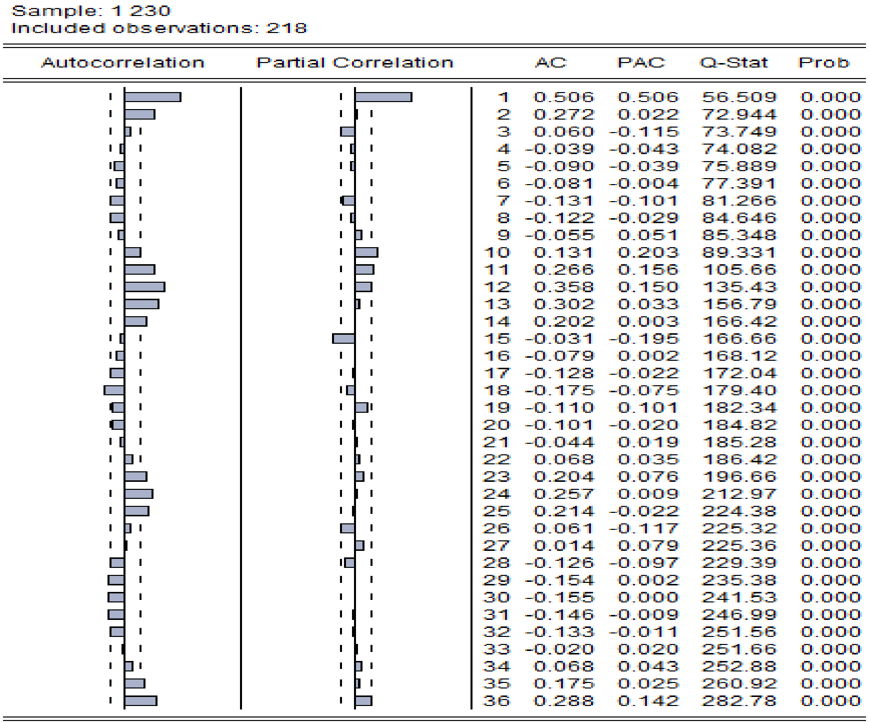


Fig. 1. Intensity and frequency of periods

Source: Author's calculations and Eviews 7

The following correlogram analysis of ACF and PACF shows periodic effects (See Fig.2) .



Sample: 1 230
Included observations: 218

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.506	0.506	56.509	0.000
		2	0.272	0.022	72.944	0.000
		3	0.060	-0.115	73.749	0.000
		4	-0.039	-0.043	74.082	0.000
		5	-0.090	-0.039	75.889	0.000
		6	-0.081	-0.004	77.391	0.000
		7	-0.131	-0.101	81.266	0.000
		8	-0.122	-0.029	84.646	0.000
		9	-0.055	0.051	85.348	0.000
		10	0.131	0.203	89.331	0.000
		11	0.266	0.156	105.66	0.000
		12	0.358	0.150	135.43	0.000
		13	0.302	0.033	156.79	0.000
		14	0.202	0.003	166.42	0.000
		15	-0.031	-0.195	166.66	0.000
		16	-0.079	0.002	168.12	0.000
		17	-0.128	-0.022	172.04	0.000
		18	-0.175	-0.075	179.40	0.000
		19	-0.110	0.101	182.34	0.000
		20	-0.101	-0.020	184.82	0.000
		21	-0.044	0.019	185.28	0.000
		22	0.068	0.035	186.42	0.000
		23	0.204	0.076	196.66	0.000
		24	0.257	0.009	212.97	0.000
		25	0.214	-0.022	224.38	0.000
		26	0.061	-0.117	225.32	0.000
		27	0.014	0.079	225.36	0.000
		28	-0.126	-0.097	229.39	0.000
		29	-0.154	0.002	235.38	0.000
		30	-0.155	0.000	241.53	0.000
		31	-0.146	-0.009	246.99	0.000
		32	-0.133	-0.011	251.56	0.000
		33	-0.020	0.020	251.66	0.000
		34	0.068	0.043	252.88	0.000
		35	0.175	0.025	260.92	0.000
		36	0.288	0.142	282.78	0.000

Fig.2. ACF and PACF , correlogram

Source: Eviews 7

Thus , models including seasonal components can be constructed as follows :

$$\Delta ипц_t = 0.7256\cos(\omega t) - 0.3137\sin(\omega t) - 0.3922\cos(2\omega t) + 0.2153\cos(12\omega t) + 0.3372\cos(13\omega t) + 0.1991\sin(37\omega t)$$

High frequency and harmonic oscillations for inflationary processes, respectively, are equal to 0.0826 and 54.

Econometric estimates of the component show that the autonomous part is statistically significant (See Table 2).

Table 2 .

Variable	Coefficients	Std . open	t- statistics	probability
Trend component (c)	100.3359	0.156755	640.0793	0.0000
Trend Component (@trend)	0.001112	0.00125	0.889897	0.3745
Residual component u(-1)	0.333744	0.064363	5.185301	0.0000

Source: Author's calculations and Eviews 7

Econometric models with a combination of Fourier series model components take the following form:

$$ипц_t = 100.4614 + 0.723093\cos(\omega t) - 0.289607\sin(\omega t) - 0.314703\cos(2\omega t) + 0.241366\cos(12\omega t) + 0.327632\cos(13\omega t) + 0.212861\sin(37\omega t) + 0.337403u_{t-1}$$

To account for economic shocks, the Fourier series models were extended by using a dummy variable ФП{0,1} as additive. Then the Fourier series expansion would be as follows . Here ФП is a dummy variable and is defined as 0 and 1.

$$\text{ИПЦ}_t = 100.4157 + 0.673452\cos(\omega t) - 0.31071\sin(\omega t) - 0.318\cos(2\omega t) + 0.260353\cos(12\omega t) + 0.317045\cos(13\omega t) + 0.140359\sin(37\omega t) + 0.244708u_{t-1} + 1.484984\ \text{ФП}$$

When comparing the quality of the models (RMSE, MAE, MAPE, TIC, Durbin-Watson statistic, Akaike info crit. Fourier series (FS) and extended Fourier series (ESF) models, it is clear that the ESF models are more significant. Results of the Dickler-Fuller tests show that the model is stationary at all levels of statistical significance .

5. Modeling of inflation processes based on SARIMAX models .

As stated above , SARIMAX models are structurally different from SARIMA models . Since SARIMAX models include exogenous variables .

When adding dummy variables taking into account economic shocks and exogenous variables , the SARIMAX model looks as follows : $(1-\emptyset_1 L)(1-\varphi_1 L^{12})\mu_t = (1+\theta_1 L)\varepsilon_t$

$$(1-0.4224L)(1-0.2420L^{12})\mu_t = (1+0.1019L)\varepsilon_t$$

$$\text{ипц}_t = c + \Theta\text{ФП}_t + \mu_t$$

$$\text{ипц}_t = 100.3304 + 2.085\text{ФП}_t + \frac{(1+0.1019L)\varepsilon_t}{(1-0.4224L)(1-0.2420L^{12})}$$

$$\text{ипц}_t = 100.3304 + 2.085\Phi D_t + \frac{\varepsilon_t \ + 0.1019\varepsilon_{t-1}}{1 - 0.24\text{ипц}_{t-12} - 0.422\text{ипц}_{t-1} + 0.1022\text{ипц}_{t-13}}$$

The results of the evaluation of the above constructed SARIMAX models provide estimates of the model parameters in the following table.

Table 4. Evaluation of the parameters of the SARIMAX models and the quality of the forecasting models .

Parameters	**SARIMAX (100) (100)[12]**	**Standard deviation**	**SARIMAX (102)(100)[12]**	**Standard deviation**
C	100.3318	[0.143681]	100.3304	[0.14932]
FP	2.060154	[0.481008]	2.085979	[0.489423]
AR(1)	0.454368	[0.063536]	0.422471	[0.064776]
SAR(12)	0.243506	[0.06088]	0.242089	[0.064191]

MA(2)		0.101964	[0.000537]
Coeff . of determination	0.426393	0.42843	
RMSE	0.316724	0.15172	
MAE	0.316724	0.15172	
MAPE	0.315524	0.15 117	
Akaike info criteria	2.516702	2.52290	
Schwarz criteria	2.581542	2.60395	
Durbin-Watson s.	2.007207	1.94838	

Source: Author's calculations and Eviews 7

When considering the forecasting results of the SARIMAX (100)(100)[12] and SARIMAX (102) (100)[12] models, it is evident that the SARIMAX (1 0 2) (1 0 0)[12] models have a price closer to the actual indicator (See Table 4). The quality indicators of the SARIMAX (1 0 2) (1 0 0)[12] models are considered more acceptable than SARIMAX (1 0 0) (1 0 0)[12]. The SARIMAX model consists of exogenous variables added to the SARIMA model. This is an integration of regression models to SARIMA models . This model combines the advantages of both models . ARIMA methods provide for autocorrelations of the residuals. Comparative forecasting graphics for SARIMAX and RRF models are described below (See Fig . 3 , Table 5):

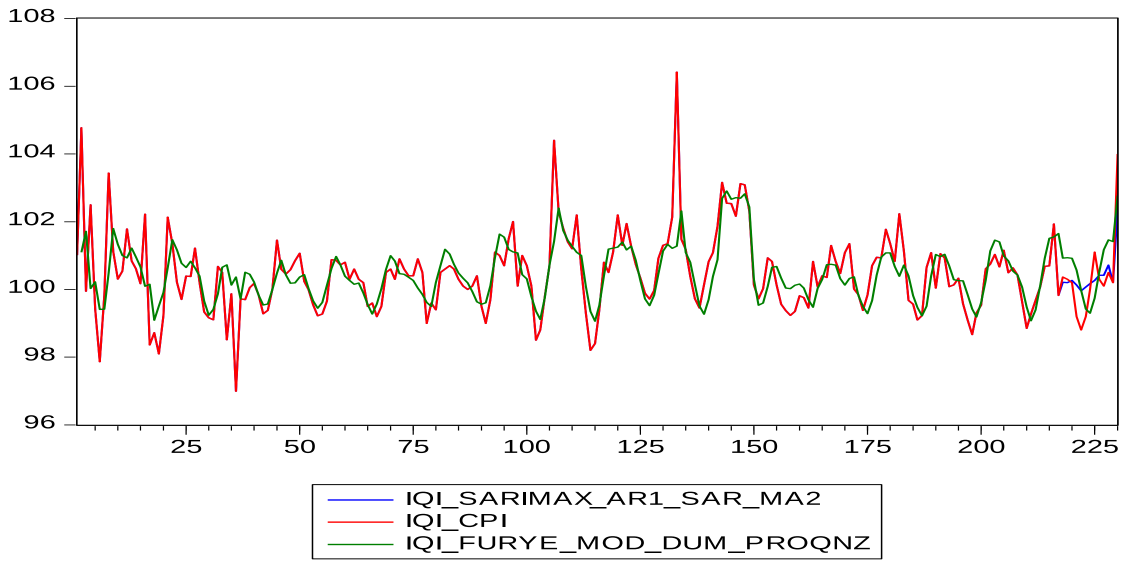


Fig. 3 . SARIMAX model prediction and Extended Fourier series

Source: Author's calculations and Eviews 7

Table 5. Evaluation of the SARIMAX model and the Fourier series

n	SARIMAX (1 0 2) (1 0 0)[12] model	RRF model	CPI
218	100.21	100.92	100.36
219	100.20	100.93	100.30
220	100.27	100.92	100.20
221	100.13	100.57	99.20
222	99.96	99.97	98.80
223	100.07	99.42	99.20
224	100.17	99.30	100.00
225	100.27	99.74	101.10
226	100.41	100.50	100.30
227	100.42	101.17	100.10
228	100.72	101.46	100.50
229	100.21	101.41	100.20
230	102.39	102.77	104.00

Therefore, in all these models and when forecasting and only then do they acquire an approximation to the actual assessment when all significant processes occurring in the economic system are formed on the basis of chance and retain their market mechanisms . These are necessary conditions for obtaining the smallest deviations in predictive models . Evolutions in the behavior of economic entities lead to inflationary processes from one quality to another. In this case , the sufficient condition is the preservation of qualities by a significant amount . These conditions increase the adequacy of the approximation of models to actual indicators .

Conclusion

Based on the results of the study, it was determined that the use of Fourier series and SARIMAX models is considered more significant. When modeling and and predictions of inflation processes in Azerbaijan. Consequently, the rate of regulation of inflation processes directed to the state of equilibrium is equal to 6 (six) months. For all these models, forecasting only then acquires an approximation to the actual assessment, when all significant processes occurring in the economic system are formed on the basis of randomness, and retain their market mechanisms. This is a necessary condition for the smallest deviations in predictive models . Evolutions in the behavior of economic entities transfer inflation processes from one quality to another. In this case , a sufficient condition is a significant preservation of qualities . These conditions increase the adequacy of the approximation of models to actual indicators .

Therefore, the application of these methods in the study of inflationary processes has theoretical and practical significance in determining macroeconomic monetary policy.

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