

# Interdisciplinary and interdisciplinary integration in mathematics teaching

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## Abstract

Improving the quality of education is always one of the priority issues. High school math teaching and related topics are always relevant. Learning to solve problems increases students' interest in the subject being studied. Various methods and tools are widely used in teaching problem solving in high school mathematics. To analyze the problem and its solution, you can change the data in the problem, find it, replace it with others, which can be viewed as a situation that increases the skill of students. Determining the dependencies between lines of content and their integration is one of the basic learning conditions. Students often have difficulty applying algebraic and geometric content strings at the same time when solving word problems. To overcome this difficulty, consideration should be given to the use of interdisciplinary and interdisciplinary integration. In solving such problems, it is important that students, in addition to theoretical material, have knowledge based on deductive reasoning. Although the teaching approach based on general pedagogical laws is the same for all subjects, each subject has its own characteristics. Given these similarities and differences, the development of interdisciplinary and interdisciplinary relationships should be seen as one of the key factors in improving the quality of education. The high school mathematics course covers many of the subjects taught in the school - physics, chemistry, geography, technology, biology, etc. are closely related. In many branches of physics, including mechanics, thermodynamics, etc. Dependencies between quantities are explained by mathematical models. In particular, there is no subject in our everyday life that in one way or another does not use mathematical concepts. Many problems in physics, chemistry and geography are solved by mathematical methods. This article discusses a methodology for solving problems associated with different content lines and based on interdisciplinary integration.

Key words: learning, integration, mathematical problem, quantity, functional dependence, motion problems, trajectory, speed, trapezoid, similarity, area, model, directional motion, angular velocity, quadratic equation.

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As a result of the reforms carried out in the education system of the Republic of Azerbaijan, new approaches are required in teaching mathematics in secondary schools. Interdisciplinary and interdisciplinary integration can be considered one of the factors influencing the quality of mathematics education. Thus, mathematical expressions, inequalities, formulas learned in mathematics are somehow connected with the solution of practical problems that exist in real life. Interdisciplinary integration plays an important role in solving practical and applied problems in teaching students these processes. The main purpose of this article is to explore the possibilities of interdisciplinary and interdisciplinary integration in problem solving learning. This article examines local and international research on the topic. At the same time, the results obtained were compared with the actual problem in the existing mathematics textbooks of the Republic of Azerbaijan.

The role of mathematics in the development of knowledge and skills of students, in the formation of personality is irreplaceable. Sustainable reforms in the education system of the Republic of Azerbaijan and the use of modern technologies in education require the use of different approaches in the learning process. Mathematics plays an important role in the practical activities of students. The importance, goals and objectives of mathematics as a main subject are related to the following:

- Mathematics is an irreplaceable tool for mental development;
- Mathematics is an important tool in the formation of personal qualities;
- Mathematics is directly related to many modern specialties;
- Mathematics and modern human life are like a holistic "organism";
- Mathematics is an integral part of world culture [2].

The development of theoretical and practical materials in the course of mathematics in secondary school is carried out mainly by solving problems.

If we analyze these goals and objectives separately, it cannot be denied that mathematics is interconnected not only with the exact, but also with the humanities. The outstanding Russian poet A.S. Pushkin praised mathematics and said: "There is beauty in music and poetry in mathematics as well." It is important to arouse students' interest in the subject for the formation and development of mathematical knowledge and skills. In the process of teaching mathematics, the creation of motivation, problem situations is carried out mainly through solving problems. The "broad" activity of a student in the learning process depends on the nature of the tasks facing him. The concept of matter, the definition given to it is unclear and unambiguous [32].

In Soviet times, high school taught subjects known as "Mathematics", "Algebra", "Geometry", "Algebra and the beginning of analysis". According to the new educational program (curriculum) of the Republic of Azerbaijan, only the subject "Mathematics" is taught in the secondary school course. Five lines of content (numbers and operations, measurements, algebra and functions, geometry, statistics and probability theory) are given within one subject. In addition to the positive elements of this approach, there are some problems in the learning process. Overcoming this and improving the quality of teaching depends on interdisciplinary and interdisciplinary integration.

Interdisciplinary and interdisciplinary communication in the learning process, its development is one of the important tasks facing modern education. The rapid development of science and technology, an increase in the volume of scientific innovations and knowledge make integrative learning relevant [3].

Among the subjects taught in secondary schools, mathematics is the leading subject and plays the role of a real object that reveals the existence of connections between other subjects.

The goals of teaching mathematics are broad and can be broadly described as follows:

- forms a way of thinking about the fact that mathematics is a method of describing and understanding reality;

- creates the idea that mathematics is an integral part of universal culture and a driving force in the development of society [3].

- creates a real basis for continuing education, learning other subjects, acquiring the necessary knowledge, skills and habits in order to apply them in practical activities;

The formation of students' heuristic activities with problem solving in the process of teaching mathematics and the conditions justifying it are generally related to teaching methods [4].

This article examines the possibilities of analysis and use of interdisciplinary and interdisciplinary integration in solving mathematical problems related to different content lines.

Professor Ə.M. In his research, Mammadov explored the role of the concepts of model and mathematical model in the teaching of computer science.

The content of the problem, the conditions for the division of the problem into "sub-problems" and the application of modeling in problem solving contribute to the development of students' logical thinking [35, 43].

Intrasubject integration involves the coordination of relevant lines of content through internal capabilities. According to the structure of the high school mathematics course, interdisciplinary integration is carried out in several stages. The implementation of integration between elements of the same content line by grade follows from the structure of the mathematics course. For example, the expansion of the concept of numbers to the concepts of natural numbers ( $\mathbb{N}$ ), integers ( $\mathbb{Z}$ ) and rational numbers ( $\mathbb{Q}$ ) is associated with the daily activities of people in real life. Demand itself makes reality a reality. Including zero (0), the inverse of natural numbers (negative integers), and fractional numbers in the sequence correlates the integration for different classes. Two models of integration (horizontal and vertical) are used for the conscious assimilation of the studied material. The connection between the content lines of the subject and the pedagogical "skill" of the teacher and the scientific potential for their formation is the basis of horizontal integration. In mathematics teaching, horizontal integration is linked between five lines of content.

In traditional curricula and mathematics textbooks, content lines have been partly included separately. In general, parallel and mixed inclusion of lines of content can be seen as an increase in learning efficiency. The existence of scientific, logical and pedagogical problems in the inclusion of lines of content depends on the structure of interdisciplinary integration [32].

Vertical integration involves planning topics (standards) on a quarterly (semi-annual and annual) basis, developing students' knowledge, skills and habits. Vertical integration should also ensure the implementation of interdisciplinary communication.

The ability to differentiate the learning process and content standards at a conscious and in-depth level is considered at different stages of learning. Here, the directions for improving mathematical programs, teaching concepts and their properties, implementing developmental and educational goals in the form of a specific system are implemented. The inclusion of the same concept in different disciplines is generalized through the synthesis of common elements and properties in the study of these concepts. The synthesis should correspond to a certain logic and help students develop logical thinking. It is necessary to analyze the existing possibilities of using

interdisciplinary communication in the formation of students' worldview, solving practical problems.

During the Soviet era, the pedagogical experiment was widely used in research on the teaching of mathematics. Some results of research conducted by mathematician A.A. A joiner in the field of "Logical problems of teaching mathematics" can be considered relevant today. Logical problems in teaching mathematics, their causes and ways of solving these problems are associated with interdisciplinary integration [40]. One should expect a unique model of disciplines that takes into account psychological and pedagogical factors in the establishment of interdisciplinary and interdisciplinary relationships. This problem can be characterized in different ways.

Problem solving problems in teaching mathematics can be divided into different aspects such as methodological, psychological and practical. Some of these problems were relevant 40-50 years ago.

For example, in the process of teaching mathematics, the process of abstraction in the context of problems to be solved in physics, chemistry and geography should not affect the physical and chemical properties of the concept. Mathematical methods are used to solve geographic problems, and the main research in this area is related to mathematics. Yu.P. Arkhipov, a researcher in the field of geography, in his research widely used mathematical methods in solving problems related to the study of the structure and geocentric state of the Earth [40].

In addition to problems solved on the basis of algorithms in traditional textbooks of mathematics, the solution of non-standard problems was also considered [11, 13, 16]. The mathematical models (geometric drawings) used to solve standard problems were easily mastered by the students.

However, in new textbooks of mathematics, written on the basis of educational programs, preference is given to solving applied problems [6, 8, 9]. The structure of these textbooks and the content of the questions proposed here are fundamentally different from traditional textbooks. Students naturally have difficulty deciding which subject to address. The emergence of this problem is associated with interdisciplinary integration. Let's look at specific questions.

Problem 1. The car covered the first half of the road at the speed  $V_1$ , and the second half at the speed  $V_2$ . Find the average vehicle speed.

It is controversial to say unequivocally that this problem is purely mathematical or physical. This justifies the formation of interdisciplinary integration with certain common concepts. It is no coincidence that scientists have identified physics as the main object of the application of mathematics. Mechanics, mathematical physics, electrical engineering, etc. The tasks of the units are solved mainly with the help of mathematical concepts. Interdisciplinary and interdisciplinary integration is carried out in the high school mathematics course mainly through problem solving. For example, consider the following problem, which implements interdisciplinary integration.

Problem 2. If the perpendicular  $NM$  is at a distance  $|AM| = x$  (variable  $x$ ) from the vertex  $A$  of the seat of an equilateral trapezoid  $ABCD$  with seats  $a$  and  $b$ , height  $h$  (Fig. 1.), find the dependence of the area of the figure  $ABNM$  on  $x$ .

This issue concerns the geometric content line. A brief description of the problem is as follows:

$ABCD$  - trapezoid  
 $AB=CD$

AD = a  
 BC = b  
 NM = h  
 AM = x  
 S<sub>ABNM</sub> - ?

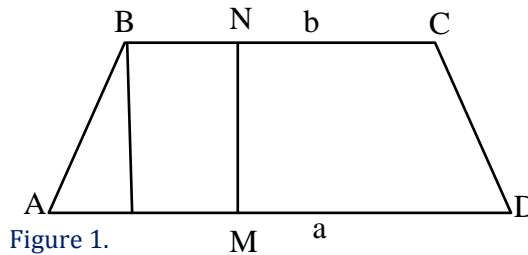


Figure 1.

The structure of the task is to link the content lines "Geometry" and "Dimensions". The area of the rectangle ABNM depends on the variable x, and depending on AM = x, the required area must be expressed differently. Depending on x, the desired area will be the areas of a triangle, quadrilateral, and pentagon. The areas of the numbers should vary depending on the functional relationship between the quantities. To answer this question, the following cases should be considered:

- 1)  $0 \leq x \leq \frac{a-b}{2}$
- 2)  $\frac{a-b}{2} < x \leq \frac{a+b}{2}$
- 3)  $\frac{a+b}{2} < x \leq a$

1. Combined expressions of content lines are transformed into mathematical "language" by applying elements of logic in interdisciplinary integration. Complete (abstract) a quadrilateral triangle for the case  $0 \leq x \leq (a-b) / 2$ .

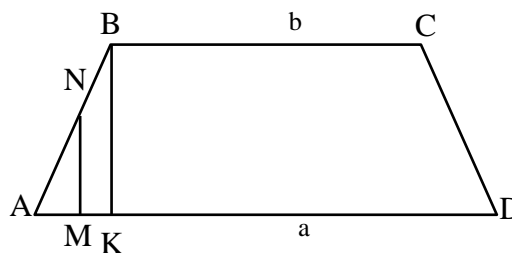


Figure 2

The geometric description of the figure allows you to find the quantity you are looking for. Regardless of the form of the image, the functional dependence is related to the content line "Numbers and operations", taking into account the properties of the trapezoid.

Let's write a brief description of the geometric figure in Figure 2.

$$AK = \frac{1}{2} (a - b)$$

$$AM = x$$

$$BK = h$$

$S_{ANM} = ?$

In Figure 2, based on the similarity of the triangles ANM and ABK ( $\triangle ANM \sim \triangle ABK$ ), we express the fraction MN with known quantities a, b, h

$MN = \frac{2hx}{a-b}$  will be taken

Since the triangle AMN is a right triangle, its area will be  $S = \frac{1}{2} AM \cdot MN$ . Hence, the area of the figure sought within a given condition

$S = (hx^2) / (a-b)$  is equal to (1).

2. Although the functional dependence belongs to the content line "Algebra and functions", the content line "geometry" plays the main role in finding the unknown quantity in the problem. Integration between the materials of the same content line (geometry) created conditions for the realization of the solution of the problem.

If we use Figure 2 for the case  $\frac{1}{2}(a-b) < x \leq \frac{1}{2}(a+b)$ , the summary of the problem would be:

$AB = CD$

$BC = b$

$AD = a$

$NM = h$

$|AM| = x$

$S_{ABNM} = ?$

According to Figure 2 and the condition, it is known that  $AK = \frac{1}{2}(a-b)$ . According to the equation  $AK + KM = x$ ,  $KM = x - AK = x - \frac{1}{2}(a-b) = (2x - (a-b)) / 2$ .

Since  $KM = BN$ , taking into account the values of the expressions AM and BN in the equation  $S_{ABNM} = \frac{1}{2}(AM + BN) \cdot h$ , the dependence of the sought quantities on x is as follows:

$$S_{ABNM} = \frac{1}{4}(4x - (a-b)) \cdot h = hx - \frac{1}{4}(a-b) \cdot h$$

$$S_{ABNM} = hx - \frac{1}{4}(a-b) \cdot h \quad (2)$$

3. The third case  $\frac{1}{2}(a+b) < x \leq a$  can be considered more broad and general for finding the quantity sought. Here, the functional relationship between condition and quantity can be described as in Figure 3. According to the description, the brief description of the issue is as follows:

$AB = CD$

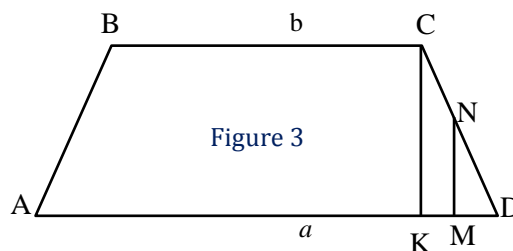
$BC = b$

$AD = a$

$AM = x$

$CK = h$

$S_{ABCNM} = ?$



The area of the figure ABCNM is equal to the difference between the area of the trapezoid ABCD and the area of the triangle NMD. Here, other concepts known from geometry are used, using the additive property of the field. When intra-subject integration is applied, concepts known to students are not given much attention. This is related to the problem in question, and the teacher should take this into account in order to make it a mini-mum. In Figure 3, the ratio KD: CK = MD: MN is paid according to the similarity sign of the triangles. Given the conditions and dependencies

The equation  $(a-b) / 2h = (a-x) / MN$  is true.

Hence, since  $MN = ("a-x") \cdot 2h / "a-b"$ , the area of the right triangle DMN will be calculated by the following expression:

$$S_{DMN} = 1/2 \cdot MD \cdot MN$$

Given that  $MD = a - x$ ,

$$S_{DMN} = ( [(a-x)]^2 \cdot h ) / (a-b).$$

Thus, the area of the figure ABCNM is determined as follows, depending on the variable x:

$$S_{ABCMN} = \frac{a+b}{2} \cdot h - \frac{(a-x)^2 \cdot h}{a-b} \quad (3)$$

Expressions (1), (2), (3) are the mathematical dependence of the quantity sought in the problem under certain conditions. In the stages of solving this problem, interdisciplinary integration was realized through content lines.

In other words, the functional dependence between quantities was expressed in a logical way based on the elements of different content lines and the corresponding descriptions between them. Thus, interdisciplinary communication plays an exceptional role in the realization of developmental and educational goals of mathematical education. Practical activities of students in the study of theoretical materials are expanding.

All three cases, taken in solving the problem under consideration, are of a theoretical nature, and B.A. This coincides with the approach suggested by Smirnov.

Consolidation of knowledge gained in the assimilation of mathematical concepts and theoretical materials is systematized by linking lines of content.

Interdisciplinary integration has always been a focus of educators and mathematicians to make learning effective by solving high school math problems. Students strengthen memory in interdisciplinary communication, feel that there is a complex connection between subjects, determine the role of mathematics in the study of other subjects. At the same time, with the help of other subjects, they master the content of mathematical concepts. Since a concept is a form of thinking that reflects the properties of the objects and events in question, it cannot be studied only with the help of mathematics and other disciplines. Each concept has its own characteristics and properties. Abstraction, which is widely used in mathematics teaching, indirectly limits or

generalizes interdisciplinary and interdisciplinary integration depending on the situation. For example, four sides of a rectangle, four right angles, and equality of diagonals are abstracted. The detection and analysis of other properties of a rectangle depends on the formulation of a specific problem. The concepts included in the lines of geometric content are translated into mathematical language and are used in physics, astronomy, chemistry, geography, etc. is used. The circle, its length and dependence on the radius are studied in the course of mathematics. The definition of a circle describes its formation. "A figure formed by a set of points equidistant from a given point on a plane is called a circle." This definition summarizes the origin (gene) of the concept and its formation. Intra-subject relationships not only develop the mathematical and logical thinking of students, but also allow them to shape their worldview. In the so-called "genetic" form of definition, abstraction is realized through logical judgment, and no mistakes are made in the formation of the concept. The knowledge and skills gained by students about the circle in the course of mathematics are also used in physics to solve specific problems. For example, uniform movement around a circle and the quantities that characterize this movement provide interdisciplinary and interdisciplinary integration.

Thus, methods of analysis and synthesis have a special place in integration. In the decisive analysis "the scheme of increasing the direction of L.M. This is reflected in Friedman's research. Here, the analysis is carried out in the "downward direction" after a certain stage [42].

The trajectory MN of an object moving from point M to point N along a circle is determined by the angle  $\varphi$  of a circle of radius  $r$ . The concepts of arc length and angle of rotation are used here. The quantity  $\varphi$ , equal to the ratio of the length of the trajectory (arc MN) of the object to the radius of the circle, is called the angle of rotation and is denoted  $\varphi = l / r$ .

In the course of mathematics, the concepts learned by the student - a circle, its radius, the length of a circle - are applied to a specific problem (physics) and ideas about this concept are expanded. If a material point is in motion, it exists with its own speed and time spent in motion. This relationship is known to students from questions related to the transition from the course of elementary mathematics. From the geometry course, the concepts of angle and rotation (movement) in physics are generalized by the concepts of rotation angle and angular velocity. The value measured by the ratio of the angle of rotation to the time spent on this rotation is called the angular velocity ( $\omega = \varphi / t$ ). The concepts of trajectory (S), time (t) and speed (V) in the process of solving mathematical problems related to motion are described in physics: motion in a circle, angle of rotation, angular velocity, period of rotation ( $T = t / N$ ,  $Nt$  - number of cycles in time) and rotation. is similar to concepts such as frequency. The acquired mathematical and physical knowledge creates conditions for studying the motion of space objects and the quantities that characterize them in senior classes.

Graphic modeling in solving some problems of motion simplifies the solution of complex problems [35].

Modeling is widely used to determine relationships between quantities and to solve motion problems. The implementation of interdisciplinary modeling is mainly carried out in interdisciplinary integration. The principle of “continuity” must be respected when including topics related to each subject for proper integration [43]. Mathematical methods are used to solve many theoretical and practical problems in physics. In general, there is no physics section in which mathematical methods and calculations are not used. There are some practical problems that make it difficult to say whether it is purely mathematical or physical: in mathematics - direct and inverse proportional quantities, a linear function and its graph, a quadratic triangle and its graph, etc. When subjects are taught in mathematics, you can look at the kinematics of linear motion in physics and the dependence of basic quantities on them. Along the trajectory characterizing the movement of an object, the study of its state, the maximum and minimum cases of displacement in time is a process based solely on mathematical methods. Mathematical description of the projection of the displacement of the body, the geometric position of the displacement graphs and time coordinates, etc. based on mathematical knowledge. For example, in mathematics, it is impossible to teach the projection of the displacement of an object with uniform motion in physics, time-dependent plots of displacement and coordinates without learning the quadratic triangle and its graphic topics.

The use of American, Canadian and Turkish educational models is preferable in mathematics textbooks written on the basis of a new program in the Republic of Azerbaijan. Reforms in education and the transition to another model create certain problems in the educational process. One of the main problems is the sequence of five lines of content in one textbook (mathematics).

The Hungarian mathematician D. Poya's research on problem solving is still relevant today, despite the fact that a lot of time has passed. Parallel study of mathematical and psychological problems of learning to solve problems was analyzed by D. Poya for a wide range of possibilities of connections between the components of the educational process [36]. The specified work of D. Poy was translated from English into Russian in 1959 under the guidance of Yu.M. Gaiduk. This manual examines the concept of the problem, the problem of its solution and determines the directions of the teacher's activity in the learning process. Giving preference to the computational method when solving problems creates conditions for the development of students' mathematical and logical thinking. Around 1954-1965, the Soviet Union preferred to use arithmetic when solving mathematical problems.

Modernization of teaching methods and technologies, successful problem solving contributes to the development of the training process.

When studying the square triangle  $y = ax^2 + bx + c$ , students actually got an idea of physical quantities by studying whether its graph is a parabola, the branches of the parabola rise or fall depending on the sign of the value of  $a$ , the coordinates of the vertex of the parabola, and the values obtained by the quadratic triangle. they happen. It is impossible to fully know the physical process without knowing the mathematical theory. In mathematics, the content of motion problems is

associated with physical quantities. Quantity is a broad concept, and the initial ideas about it are formed in the course of mathematics in elementary school. For example, it is wrong to say that the concepts of path, time, and speed are specific mathematical and physical quantities. Mathematical and physical concepts are used together to solve motion problems. As a result, physical concepts are analyzed based on mathematical information.

Problem 3. The object throws up with an initial speed of 30 m / s. How many seconds will the object reach a height of 25 m?

To solve this problem, you need to choose a model. When modeling a real object, a physical quantity is converted into a mathematical object (symbol). Here, the height at which a vertically thrown object is located in  $t$  seconds, regardless of air resistance, is calculated using the well-known formula  $H = V_0 t - (gt^2) / 2$ . Mathematically, the object's trajectory describes the graph of a square triangle. That is, the trajectory of the object relative to the parabola is being studied. Here the "arms" of the parabola should point up. Mathematically, it can be assumed that an object thrown at two different values of  $t$  will be 25 m above the ground. Before physically solving this problem, it is necessary to obtain diagnostic information related to the solution. Solving problems in this way, students not only get acquainted with theoretical material, but also visually feel interdisciplinary integration.

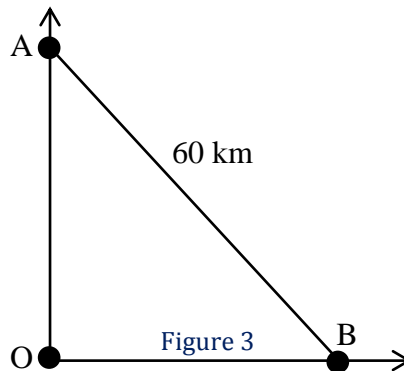
In high school math, you might run into problems with movement, since the distance between two moving objects also depends on the angle between the directions they move. In tasks of this type, data is modeled and used in elements of the geometric content line. Trigonometry is widely used to solve such problems. The relationship between the elements and angles of a triangle is related to the characteristic numbers of moving objects. In the high school mathematics course, a special place is given to cases of changes in the distance between objects moving in a direction perpendicular to each other from the same point. In solving such problems, the integration between lines of content is related to other subjects. Let's look at some examples.

Issue 4. Two ships simultaneously left the port to the north and east. After 2 hours, the distance between them was 60 km. If the speed of one of the ships is 6 km / h faster than the speed of the other, find the speed of each of them.

In this case, you should establish interdisciplinary integration and repeat topics for lines of content. The situation described in the issue is as follows.

- the movement took place simultaneously and the trajectory of movement was realized on mutually perpendicular straight lines;
- 2 hours after departure, the distance between ships is 60 km;
- the speed of one of the ships exceeds the speed of the other by 6 km / h;
- What are the speeds of the ships?

The geometric model of movement can be described as follows.



Suppose that the speed of one of the ships is  $x$  km / h. Then the speed of the other ship is will be  $(x + 5)$  (km / h). According to the Pythagorean theorem, the mathematical model (equation) for finding the desired value will be as follows:

$$[(2x)]^2 + [(2(x + 6))]^2 = [60]^2 \quad (1)$$

If we simplify equation (1), we get the equation  $x^2 + 6x - 432 = 0$ . Here  $x = -24$  and  $x = 18$ . Of these, only  $x = 18$  satisfies the condition of the problem. This means that one of the ships has a speed of 18 km / h, and the other - 24 km / h. When solving such problems, the teacher should try to create conditions for the complex application of links between lines of content.

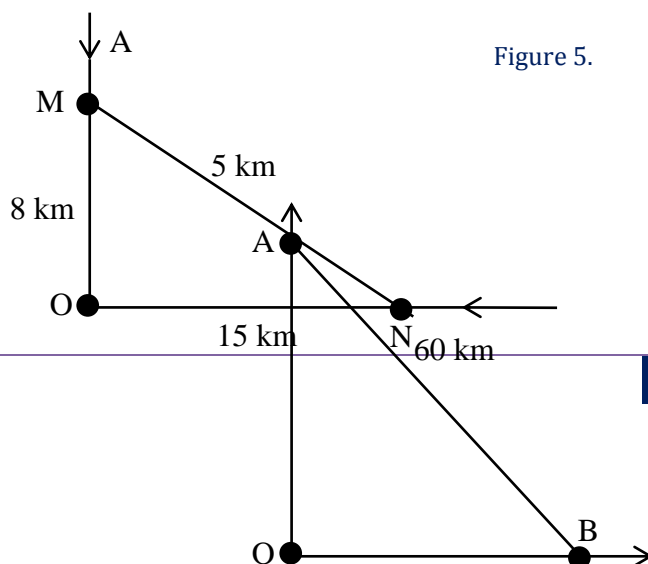
Issue 5. The cyclist and the motorcyclist moved towards the intersection of two mutually perpendicular roads. At some point, the cyclist was 8 km and the motorcyclist 15 km to the intersection. If the speed of the cyclist is  $1/3$  km / min and the speed of the motorcyclist is 1 km / min, how many minutes will the distance between them be 5 km?

This release belongs to the section on elevated issues. Having considered all kinds of movement problems in high school mathematics, solving such problems can be considered the final result. The situation in this question is as follows:

- cyclists and motorcyclists move to the intersection on the road perpendicular to each other;
- the distance from the intersection to the cyclist is 8 km, and to the motorcyclist is 15 km;
- the speed of the cyclist is  $1/3$  km / min, and the speed of the motorcyclist is 1 km / min;
- How long will the distance between them be 5 km?

In this case, the relationship between the quantities is clearly indicated, as well as the space and direction of movement of each object. Thus, students have many opportunities to create a complete picture of the movement.

The geometric model describing the movement can be represented as follows.



The main difference between this number and number 1 is that it reflects the direction of travel. Here, depending on time, the distance decreases relative to the point O. There is a moment of movement when the length of the MH segment is 5 km. Let us assume that the distance between the cyclist and the motorcyclist after the time  $t$  is 5 km. Then, depending on the state of the problem, the cyclist will be at a distance from the point O  $(8 - \frac{1}{3}t)$  (km / min) and the cyclist  $(15 - t)$  (km / min) during this period. The hypotenuse (distance between them) of the right-angled triangle describing this moment can be expressed by the following equation within the limits of the condition that we accept according to the Pythagorean theorem.

$$(8 - \frac{1}{3}t)^2 + (15 - t)^2 = 25$$

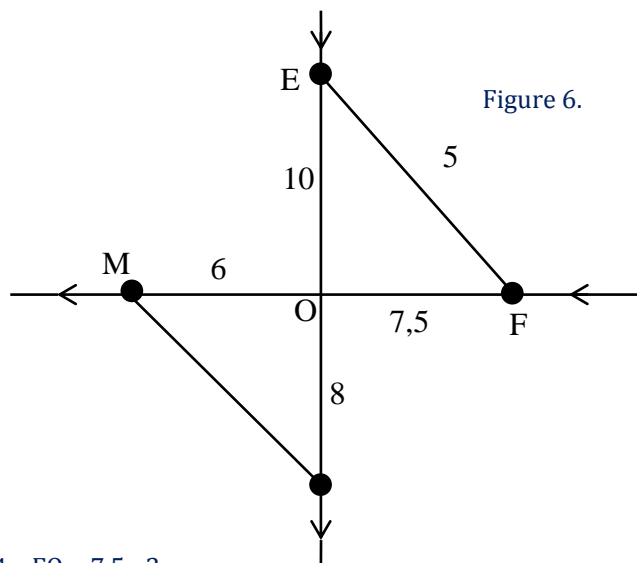
After performing certain transformations in this equation, the quadratic equation is  $5t^2 - 159t + 1188 = 0$ . The root of the equation is  $t = 12$  or  $t = 19.8$ . The distance between the cyclist and the motorcyclist can be 5 km in 12 minutes or 19.8 minutes, since both roots satisfy the corresponding equation and condition of the problem. Here 19.8 minutes can be written as  $194/5$  minutes = 19 minutes 48 seconds. Here the distance between the tourist and the cyclist at the moment  $t = 12$  minutes is 5 km, which corresponds to the fact that both of them are located in the positive direction of the coordinate axes.  $t = 18.9$  m / min means that they move in the negative direction of the coordinate axes. In general, specifying the numerical values of quantities in a way that students understand increases their interest in solving problems. Care should be taken when changing the units of measurement of quantities or when converting them to the same unit, so that lengthy calculations do not lead to erroneous results.

**Problem 6.** A tourist at a speed of 4 km per hour was moving from north to south, and a tourist at a speed of 3 km per hour was driving from east to west along a mutually perpendicular road. A tourist to the north is 10 km from the intersection of perpendicular roads, a tourist to the east is 7.5 km. What is the distance between these tourists 10 km?

To solve this problem, you can use the approach to Problem 5. However, the resulting root of the equation does not satisfy the condition of the problem with this approach. Let's look at the solution.

Suppose that in  $x$  hours the distance between the tourists is 10 km. Then the distance from the north to the junction 10 is  $4x$  (km), and the distance from the east to the junction 7.5 is  $3x$  (km). If we apply the Pythagorean theorem here, we get the equation  $(10 - 4x)^2 + (7.5 - 3x)^2 = 100$  (2). If we solve this equation, we get  $x = 1/2$  or  $x = 4.5$ . This means that the distance between tourists can be 10 km in  $1/2$  hour or 4.5 hours. For  $x = 4.5$ , the values of the expressions  $10 - 4x$  and  $7.5 - 3x$ , accepted in the problem, are expressed in negative numbers. However, since equation (2) includes the squares of the expressions  $10 - 4x$  and  $7.5 - 3x$ , there is no mathematical error in the problem. The question arises: how to give a geometric description of the result?

This question encourages students to do research, and the problem can be divided into two "sub-tasks". When  $x = 4.5$ , the absolute value of the expression  $10 - 4x$  is  $10 - 4.5 \cdot 4 = 8$ , and the absolute value of the expression  $7.5 - 3x$  is  $7.5 - 3 \cdot 4.5 = 6$ . Here are the numbers kilometers traveled by tourists after crossing roads 8 and 6, expressed in kilometers. Having analyzed in this way, the situation described in the issue can be expressed as follows.



In Figure 6,  $EO = 10 - 4x$ ,  $FO = 7.5 - 3x$ .

Comparison of Problem 5 and Problem 6 shows that on a straight line perpendicular to each other, the distance after crossing the road can be equal to the distance between them at a certain moment. The geometric explanation for this is that in a rectangular coordinate system, equal (equal) distances can be found in any quarter. This is possible at different point coordinates. However, in this case, the solution is unique under the given condition, and both conclusions are correct.

Issue 7. Two tourists set off from points A and B at a constant speed. Having reached the last point, they immediately returned at the same speed. Their first meeting was 16 km from point B, and 8 hours after the first meeting, the second meeting was 12 km from point A. Find the distance between points A and B and the speed of the tourists.

At the same time, the distance between stations and the speed of movement of tourists are not clearly indicated. However, the dependencies between the quantities specified in the problem are specified indirectly in the form of their ratio. When solving such problems, it is necessary to try to divide this problem into subtasks, adding some descriptions. With this approach, students can generalize by referring to didactic patterns from simple to complex, from simple to complex. Suppose the distance between points A and B is  $S$ , and the tourist speeds are  $V_1$  and  $V_2$ , respectively. The problem situation can be modeled as follows.

Figure 6.

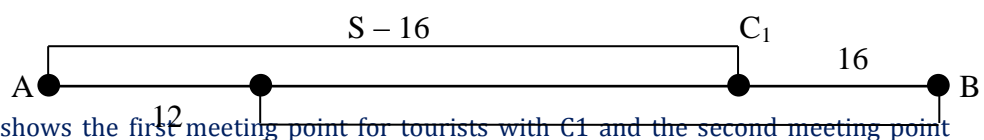


Figure 7 shows the first meeting point for tourists with  $C_1$  and the second meeting point with  $C_2$ . Depending on the state of the problem and the known model, the mathematical relationships between the quantities can be written as follows.

$$\begin{cases} \frac{S-16}{V_1} = \frac{16}{V_2} \\ \frac{S-16+12}{V_2} = \frac{S-12+16}{V_1} \end{cases} \quad (3)$$

(3) The system of equations can be written as follows.

$$\begin{aligned} \frac{S+4}{S-4} &= \frac{V_1}{V_2} \\ \frac{S-16}{16} &= \frac{V_1}{V_2} \end{aligned} \quad (4)$$

Based on equations (4), we get the equation  $(S + 4) / (S - 4) = (S - 16) / 16$ . If we solve this equation

$S^2 - 36S = 0$ ,  $S = 36$ . It is known that the second meeting lasted 8 hours. After the first meeting, the tourist from point A covered the distance  $(s + 4)$  km to the second meeting. That is, the equation  $S + 4 = 8V_1$  is correct. Since  $S = 36$ , we get  $V_1 = (36 + 4) / 8 = 5$ . Given the value of  $V_1$  in (4),  $V_2 = 4$ .

So, the distance between points A and B is 36 km, and the speed of tourists is 5 km / h and 4 km / h, respectively. You can always somehow establish a connection between the lines of content when solving various content issues related to traffic. In general, solving such complex problems completes the solution of all sorts of problems related to movement. Thus, theoretical and practical knowledge gained by students in this process can be used in physics.

The solution of many physics problems requires deep mathematical knowledge from students. Although the physical nature of some physics problems is known to students, they have difficulty in applying some mathematical apparatus. Solving this type of physics problem is closely related to mathematical concepts and requires the teacher's ability to incorporate additional concepts.

When solving some physical problems, it is necessary to solve a system of equations with several variables. For example:

"The car starts off with a constant acceleration  $a_1$ , and after reaching a certain speed  $v$  it moves at a constant speed for a certain period of time and stops with a constant acceleration  $a_2$ . If the distance traveled by the car is  $S$ , determine the travel time  $t$  of the car. "

The solution to this problem is reduced to solving a system of seven equations with seven variables, and the solution consists of the following:

$$t = t_1 + t_2 + t_3, S = S_1 + S_2 + S_3,$$

$$S_1 = \frac{a_1 t_1^2}{2},$$

$$S_2 = vt_2$$

$$S_3 = vt_3 - \frac{a_2 t_3^2}{2},$$

$$v = a_1 t_1,$$

$$v - a_2 t_3 = 0$$

Here  $t_1$  is the time spent on emergency movement,  $t_2$  is the time spent at equal speed,  $t_3$  is the time spent on stopping (movement during braking),  $S_1$  is the time spent on emergency movement,  $S_2$  is the time spent on equal speed,  $S_3$  - time taken to stop,  $S_3$  - time taken to stop. distance traveled (braking movement),  $t$  - total

time spent in action. Such problems are usually solved graphically in physics lessons [26]. When solving some mathematical problems, performing complex arithmetic operations is inefficient in terms of time and computation. When solving such problems, it is advisable to use a graphical method. Preference should be given to students' independent choice when using an effective and simple method of solving the problem.

In this article, we have conducted a comparative analysis of research related to interdisciplinary integration in problem solving learning. Experience shows that there is no perfect education system and it is always important to improve learning. The following research findings should be considered:

1. It can be noted that learning to solve problems is an invaluable tool, analyzing the scientific and methodological literature on the problem, analyzing the subjects that make up training in the framework of the secondary school course. The formation of the students' outlook and their individual development is in one way or another connected with mathematical education.

2. For the rapid development of science and technology, an increase in the sources and volume of information, the correct transfer of scientific innovations to the next generation, the use of traditional approaches in teaching in itself cannot be considered acceptable.

3. Transferring the content of the problem from real life, selecting students by age and level of knowledge, along with increasing their creative activity, develops their mathematical and logical thinking.

4. There are certain problems with the provision of textbooks for the high school mathematics course under the name of a single textbook - "Mathematics" and coverage of five lines of content. The principle of "balance" should be observed when including the meaningful lines mentioned in the mathematics textbooks for classes.

5. The inclusion of non-experimental and non-research theoretical materials in the high school mathematics course is not allowed.

8. Take into account a combined approach to ensure interdisciplinary integration in mathematics, physics and chemistry programs and textbooks.

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