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Modeling and Interpreting the Dynamics of Volatility in Global Commodity Markets: An Empirical Application of GARCH, TGARCH, and EGARCH Frameworks to West Texas Intermediate (WTI) Crude Oil Prices (2008–2024)

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Abstract

Global commodity markets, particularly crude oil markets, are inherently characterized by pronounced price volatility driven by geopolitical tensions, macroeconomic shocks, financial speculation, and structural changes in energy demand and supply. Accurately modeling and forecasting such volatility is essential for policymakers, investors, and risk managers seeking to design effective stabilization, hedging, and investment strategies. This study provides a comprehensive empirical investigation of oil price volatility by applying three prominent members of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family—namely the standard GARCH, Threshold GARCH (TGARCH), and Exponential GARCH (EGARCH) models—to monthly West Texas Intermediate (WTI) crude oil prices over the period from January 2008 to June 2024. The selected timeframe captures multiple episodes of extreme market turbulence, including the global financial crisis, oil price collapses, the COVID-19 pandemic, and recent geopolitical disruptions. The analysis proceeds through a structured econometric methodology, beginning with the transformation of price levels into return series, followed by stationarity testing, descriptive statistical analysis, and volatility modeling. Empirical results reveal strong volatility clustering and persistence in WTI oil returns, confirming the suitability of GARCH-type models for capturing oil price dynamics. While asymmetric specifications (TGARCH and EGARCH) provide valuable insights into the differential effects of positive and negative shocks, the findings indicate that the GARCH(1,1) model offers the most robust performance in terms of volatility extraction and persistence representation for the studied period. Overall, this study contributes to the growing literature on energy market volatility by offering updated empirical evidence on the effectiveness of GARCH family models in modeling crude oil price fluctuations, with important implications for risk management, forecasting accuracy, and energy-related economic policymaking.

JEL Classification: C22; Q41

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I. Introduction

Oil is a fundamental commodity in the global economy, serving as the primary source for output generation and supporting economic development. Oil prices significantly influence the macroeconomics of various countries, with fluctuations commonly referred to as oil shocks, exerting notable impacts on key macroeconomic variables including economic growth, inflation, exchange rates, and public budgets. To anticipate and estimate oil price fluctuations and forecast future volatility, various time series models incorporate these oscillations and fluctuations. Among these models, two types stand out: Autoregressive models conditioned on variance heterogeneity, known as GARCH, which serve as effective tools for assessing the volatility and deviations impacting oil prices. The second type comprises modified GARCH models, which account for the non-linear or asymmetric effects of oil price fluctuations. In accordance with this objective, this study will assure the significance of employing GARCH family models—specifically GARCH, EGARCH, and TGARCH—to examine and quantify the volatility of WTI oil prices spanning from January 2008 to June 2024. The main question can be formulated as follows: Can GARCH models effectively model and analyze the volatility of WTI crude oil prices?

This study systematically applies econometric modeling techniques to analyze and explain the behavior of oil price volatility prevailing in global markets. The study is therefore divided into two parts. The first part of the paper is devoted to the theoretical aspect and provides an a priori description of volatility models. The second part of the research includes the applied aspect, which comprises a series of stages. The first stage concerns the calculation of the return series, the next stage is dedicated to studying the characteristics of the time series, and the last stage concerns the modeling process.

II. Background on Volatility Models

Linear time series models are widely recognized as the most utilized and applicable models for various phenomena. These models hinge on the fulfillment of specific conditions, with the stability of random residual variances over time being paramount. However, achieving these conditions proves challenging, particularly in time series related to the prices of diverse goods and services markets, which are characterized by significant fluctuations. Consequently, researchers have explored alternative models to simulate such data, leading to the emergence of several autoregressive models conditioned on variance heterogeneity. Notably, among these models are the GARCH family models, specifically GARCH, EGARCH, and TGARCH models.

1. GARCH Model (p, q)

The significance of pivotal factors influencing the trajectory of these time series becomes evident when analyzing the statistical characteristics of financial time series, encompassing heightened volatility, heavy-tailed unconditional distributions, and autocorrelation irregularities. GARCH models strive to replicate market behavior by statistically handling returns and their increased volatility. Bollerler introduced the GARCH model in 1986, defining it as an extension of autoregressive conditional variance, expressed by the following relationship (Alexander & Simon, 2018):

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \, \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

Where: $\omega\,\beta_j\,.\alpha_i$constants, σ_{t-j}^2 expected variance value at an earlier day composed of GARCH, $\varepsilon_{t-i}^2$the square of the residuals from the mean equation composed of. ARCH, i.e., the expected conditional variance of the model depends on a prior representation of the squared errors as well as a prior representation of the variable itself (Ghassan & Alhajhoj, 2012).

2. EGARCH Model

Nelson introduced this model in 1991. Within this framework, the conditional variance is contingent on both the sign and magnitude of the previous error term representation. Given that the dependent variable in this model is the logarithm of the conditional variance, it adheres to the conditions of the ARCH model, ensuring that the model parameters are positive. The EGARCH (1,1) model is described by the following relationship (Nelson, 1991):

$$\log(h_t^2) = \omega + \sum_{j=1}^{q} \beta_j \log(h_{t-j}^2) + \sum_{i=1}^{p} \alpha_i \left[\left| \frac{\varepsilon_{t-i}}{\sqrt{h^2{}_{t-i}}} \right| - \sqrt{\frac{2}{\pi}} \right] + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}^2}}$$

Where: ω refers to the constant term in the variance equation, ε_t: measures the error term, α_i: measures the ARCH effect, β_j: measures the GARCH effect, and γ_k: measures the asymmetric effect due to leverage. The asymmetry of the shocks is tested. With the following null hypothesis: $\gamma = 0 \leftarrow H_0$The impact of negative and positive shocks on volatility is identical (no difference).

3. TGARCH Model

This model was introduced by Zakoian and Rabemananjara in 1991. Within this model, the previous random error term representation is discretized based on its sign, resulting in varied levels of volatility contingent upon the shock signal and its magnitude. The following equation delineates the TGARCH (1,1) model (Cai & Stander, 2019):

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j h_{t-1}^2 + \sum_{k=1}^{k} \mu_k \varepsilon_{t-k}^2 I_{t-k}$$

Where I_{t-1} is a dummy variable that takes the value 1 when $\varepsilon_{t-i} < 0$ and 0 when$\varepsilon_{t-i} \geq 0$. The asymmetry of the shocks is tested using the following null hypothesis: $\alpha = 0 \leftarrow H_0$The effects of negative and positive shocks on volatility are symmetric (no difference).

By introducing various statistical models, studies have increasingly expanded to include the dynamics of volatility in commodity markets, especially oil prices. The following table provides a brief review of some of the key studies that have applied GARCH, TGARCH, and EGARCH models to study oil price volatility.

Table 01: An Overview of the Reviewed Resources

Author	Purpose	Model Study	Summary points
(Gunay & Khaki, 2018)	This study attempted to model the volatility of natural gas, Brent crude oil contracts, and heating crude oil contracts under different distributions.	GARCH and APARCH Models	The results regarding the estimation of the volatility of each variable by GARCH and APARCH models under gev, gat, and alpha-stable distributions, and the application of various bar, Gaussian, historical, and changed (Cornish-Fisher) VaR analyses, show that the

			APARCH model significantly outperforms the GARCH model and the fat distribution outperforms in modeling the fat tail in returns. It is expected from the results that the volatility levels estimated under the GAT distribution are significantly higher than those shown by the normal distribution.
(Halkos & Tsirivis, 2019)	The study provides a comprehensive investigation of energy price volatility.	GARCH Model and Markov-Switching GARCH Method	The study effectively showed that energy prices are so volatile that such volatility requires efficient risk management strategies and that GARCH models provide such modeling and forecasting capabilities for price dynamics. Some argued there was no single model or method that, when applied, could outperform all other models in modeling and predicting price changes for all major energy commodities.
(Chun , Cho, & Kim, 2019)	The study compares different volatility models to assess their effectiveness in predicting crude oil price movements.	Stochastic Volatility (SV), GARCH and diagonal BEKK Model.	The empirical results show that the SV-based hedging strategy outperforms the GARCH and BEKK models in terms of variance minimization, and the results are also consistent for different hedging periods. Interestingly, although accurate variance and covariance estimates are important when constructing minimum variance portfolios, we find that minimizing the main square and mean absolute error does not guarantee better hedging performance.
(Fałdziński, Fiszeder, & Orzeszko, 2020)	The study compares two different approaches to volatility prediction, depending on the proxy used to measure volatility.	GARCH model and supports vector regression (SVR).	The study found that when using daily quadratic returns as a measure of valuation volatility, SVR with properly specified hyperparameters can produce lower prediction errors than the GARCH model. If we apply the Parkinson's estimator (a more accurate approximation of volatility), the results are in favor of the GARCH model.
(Zhang & Zhang, 2023)	This paper focuses on the smooth and sharp structural changes in the volatility of crude oil prices.	FFF-GARCH and MRS-GARCH Models.	Experimental results show that a flexible Fourier form (FFF) GARCH model can accurately simulate structural changes and perform better than the traditional GARCH model in terms of fitting and prediction. The Markov switching system (MRS) GARCH model has better fitting performance than the single-system GARCH model, but it is not necessarily better than similar models in terms of prediction. Finally, the FFF-GARCH model outperforms the MRS-GARCH model in predicting crude oil price volatility and portfolio performance.
(Virbickaitė , Nguyen , & Tran, 2023)	This study explores the benefits of incorporating thick-tailed innovations, asymmetric volatility response, and extended information sets in crude oil yield modeling and forecasting.	Stochastic volatility model (SV), GARCH, and generalized	It can be concluded from the results that while the inclusion of exogenous variables, especially relevant financial and macroeconomic news, in GARCH-type models leads to a significant improvement in forecasting performance, such

		autoregressive scores (GAS).	inclusion in GAS and SV-type models only slightly improves their performance. Among the GAS family models, the latter is the most efficient in terms of in-sample fit and out-of-sample forecasting accuracy, as well as predicting the level of risk and the expected deficit.
(Zhang, Chen, & Bouri, 2023)	This research proposes a new approach for modeling and predicting crude oil volatility by integrating two time-varying densities (a state-dependent process and a Hawkes process)	GARCH-Jump model	In-sample and out-of-sample analyses show that including jump strength as an explanatory variable significantly improves the forecasting accuracy of WTI and Brent volatility. For WTI volatility, the more complex the jump strength model is, the better its predictive power. The picture is different from Brent's volatility, suggesting that the non-linear nature of volatility provides a more informative forecast.
(Geng & Wang, 2024)	This study attempted to predict the volatility of oil futures using a conditional autoregressive heteroscedasticity (GARCH) model and its extended models.	GARCH and GJR-GARCH Models	Nine econometric models, including two univariate and seven multivariate models, are compared in terms of predictive performance. The empirical results show that simple inverted models outperform multivariate models in predicting fundamental volatility, as evidenced by the model confidence set (MCS), which shows that inverted models provide better statistical accuracy in forecasts.

Source: Compiled by researchers based on the reviewed resources

III. The Empirical study

In this section, the volatility of WTI oil prices will be examined using GARCH models. To accomplish this, we will proceed through the following stages: a stability analysis of oil price returns, a descriptive examination of the oil price returns series, and the modeling of oil returns volatility.

1. Data Collection and Methodology

This study utilizes the monthly closing price data of WTI oil to analyze its volatility from January 2008 to June 2024, encompassing 198 observations sourced from the database (investing.com). WTI was selected as a global benchmark for oil pricing.

The data comprises a time series of monthly closing prices. Prior to analysis, this time series must be transformed into returns series, defined by its stability and volatility around the mean, according to the following mathematical formulation (Narayan & Seema, 2007).

$$R_t = \ln(P_t) - \ln(P_{t-1})$$

Where R_t is monthly oil yield for month t, P_t is oil closing price for month t, $P_{t-1}$is previous oil closing price for month t. This transformation stabilizes the variance and helps model returns more effectively.

Statistical software like R's "arch" library are employed to estimate GARCH, EGARCH, and TGARCH models, assessing their capability to capture the impacts of volatility and leverage on typical oil returns.

2. Testing Stationarity of Oil Price Returns Series

The unit root φ stands as the primary factor contributing to non-stationarity in the time series, with unit root tests serving as potent tools for detecting stationarity. In scrutinizing the stationarity of the WTI oil returns series, this study employed the Augmented Dickey-Fuller test (ADF), the Phillips-Perron test (PP), along with the KPSS test.

The outcomes of the tests presented in Table 02 reveal that the ADF and PP test statistics for returns fall below the critical values at a 5% significance level, indicating the rejection of the null hypothesis of non-stationarity in both tests. The KPSS test value is entirely below the tabulated value at a 5% significance level, thereby leading to the inability to reject the null hypothesis of stationarity. Consequently, the series of returns for WTI Oil is deemed stable, rendering it suitable for dependable time series analysis, forecasting, and modeling.

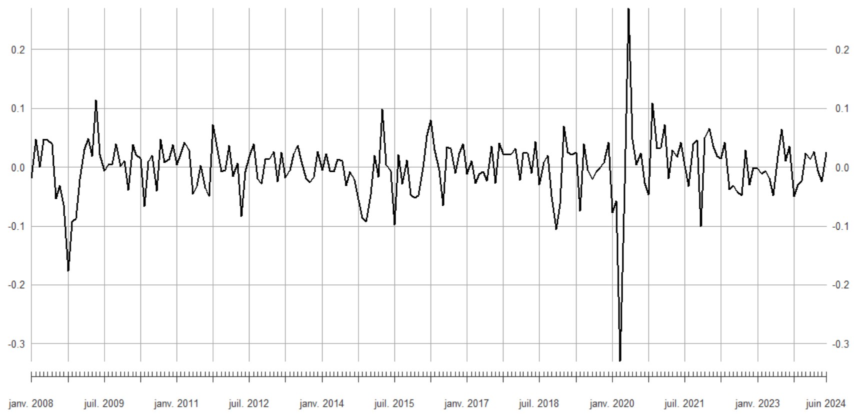
Table02: Unit Root tests Results

Test	Statistic	Lag/Truncation Lag	p-value	Null Hypothesis
Augmented Dickey-Fuller	-6.2404	5	0.01	Non-stationary
Phillips-Perron	-140.58	4	0.01	Non-stationary
KPSS	0.045724	4	0.1	Stationary

Source: Generated by researchers using the time series package in R

The stationarity of the WTI oil returns series is evident in the graph displayed in Figure 01, showcasing a relatively stationary trend converging around zero (the horizontal axis). Nonetheless, random oscillations of differing amplitudes are observed, alternating between phases of stability and subsequent stability periods, as well as between intervals of high volatility followed by prolonged periods of heightened volatility. This pattern indicates the presence of clustered fluctuations within the series.

Figure 01: WTI Oil Yield Series



Source: Generated by researchers using the time series package in R

3. Data Description

To identify the most important descriptive statistical indicators of the studied data series, the following table was prepared:

Table 03: Data Description of the Used Series

Statistic	Mean	Dev.Std	Skewness	Kurtosis	Jarque-Bera	p-Value
Value	-0.000297	0.051294	-0.8754467	14.436773	1104.4	0.0000

Source: Generated by researchers using the moments and series package in R

From the descriptive statistical indicators presented in Table (03), it is evident that the average WTI oil returns throughout the study period was negative, standing at 0.000297. The findings indicate moderate fluctuations in WTI oil returns, as illustrated by the standardized values. The standard deviation is 0.051294, and the negative skewness coefficient signifies a left-skewed distribution shape, implying a higher likelihood of negative returns over positive ones. Furthermore, the distribution of returns displays a pronounced trough and a sharp peak, indicative of outliers, as highlighted by a Kurtosis coefficient exceeding the normal distribution threshold of 3. This departure from a normal distribution is reinforced by the substantial Jarque-Bera statistic value of 1104.4, with the associated probability value falling below the significance level (0.05>Prob).

4. Modeling the Volatility of WTI Oil Returns Using GARCH Models

Model estimation is predicated on the method of maximum likelihood, if errors conform to a specific distribution, as represented by the normal distribution, the student's distribution, and the Generalized Error Distribution (GED).

4.1. ARMA Model Estimation of WTI Oil Returns

As an initial step, the WTI crude oil returns series will be estimated using the ARMA model through the maximum likelihood method employing the BHHH algorithm. The outcomes presented in Appendix (1) indicate that the ARMA (1,1) model best captures the behavior of returns series. The AR (1) coefficient of -0.1698 suggests a moderate negative correlation within returns series, indicating that current returns are inversely related to past returns. Moreover, the MA (1) coefficient of 0.4084 signifies short-term stability or momentum. The residual variance, log-likelihood value, and AIC value collectively support the appropriateness of the ARMA (1,1) model in capturing returns volatility.

Nevertheless, upon examining the impact of ARCH-LM on the residuals derived from the ARMA (1,1) model, the results outlined in Table (4) reveal that the p-value associated with the computed ARCH-LM statistic falls significantly below the predetermined significance level of 0.05. Consequently, we dismiss the null hypothesis positing the absence of an ARCH effect, indicating a lack of uniformity in the conditional variances of returns—thus suggesting variance discrepancies. This heterogeneity underscores the presence of volatility and fluctuations in the returns of West Texas Intermediate oil. Therefore, the proposed ARMA (1,1) model proves inadequate in fully capturing the intricate dynamics of oil returns, necessitating the adoption of more sophisticated models capable of accommodating the heterogeneity in conditional error variances, such as the GARCH family models, notably TGARCH and EGARCH, alongside traditional GARCH models.

Table 04: ARCH Test Results for the Residuals of the ARMA (1.1) Model Estimation

Statistic	Chi-squared	Degrees of Freedom (d.f.)	p-Value
Value	41.264	5	0.0000

Source: Generated by researchers using the FinTS series package in R

4.2. Estimating the GARCH Model for the WTI Oil Returns Series

After numerous iterations involving different versions of the GARCH model, as presented in Appendix 2, it became evident that the GARCH (1,1) model serves as the most suitable refinement for the residuals stemming from the ARMA (1,1) model. This model adeptly captures the fluctuations in WTI oil returns, with all model parameters (ω .α_1.β_1) proving statistically significant at the 10%, 5%, and 1% levels of significance, respectively.

The model's efficacy in capturing the ARCH effect on returns is discernible through various diagnostic assessments, including the Ljung-Box test, ARCH test, and Nyblom stability test (Appendix 3). The GARCH (1,1) model effectively encapsulates the volatility dynamics within the oil return series.

Moreover, the GARCH (1,1) model provides insights into the persistence of volatility, as indicated by the sum of coefficients ($\alpha_1 .. \beta_1$) nearing 0.999, suggestive of a high level of volatility persistence in WTI oil returns. Notably, the substantial ARCH coefficient (α_1) of 0.519812 in contrast to the smaller β coefficient (β_1) of 0.479188 implies that market events or shocks prompt significant volatility responses, underscoring the oil market's sensitivity to new information.

4.3. Estimating the EGARCH Model for the WTI Oil Return Series

After numerous iterations aimed at model estimation, the findings presented in Appendix 4 indicate that the EGARCH (1,1) model emerges as the optimal refinement for the residuals originating from the ARMA (1,1) model, with all model parameters ($\omega .\alpha_1 .\beta_1 .\gamma$) demonstrating statistical significance.

Comprehensive diagnostic evaluations, including the Ljung-Box test, ARCH test, and Nyblom stability test (refer to Appendix 5), affirm that the EGARCH (1,1) model proficiently captures the volatility dynamics inherent in the WTI oil returns series, showcasing an absence of notable ARCH effects or errors.

The distinctive feature of the EGARCH model lies in its capacity to assess the impact of leverage on WTI oil returns, as evidenced by the negative coefficient of ARCH (α_1) signifying its effect on returns. This observation suggests that adverse news (negative shocks) within the oil market incite or intensify volatility to a greater degree than positive news (positive shocks). Moreover, the significant γ parameter, alongside the statistically significant $\beta_1$coefficient, underscores that historical information within the oil market directly influences volatility, with its proximity to 1 indicative of prolonged memory in variance (as detailed in Appendix 4).

4.4. Estimating the TGARCH Model for the WTI Oil Returns Series

After several attempts aimed at estimating this model, the results presented in Appendix 6 reveal that the TGARCH (1,1) model represents the most appropriate refinement of the ARMA model residuals. Subsequent estimation of the TGARCH (1,1) model for the WTI oil returns series indicated that while the parameters within the average equation ($\mu .\mathrm{ra}_1 .ma_1$) prove insignificant, suggesting stable average returns, the ARCH (α_1) and GARCH (β_1) parameters stand out as positive and statistically significant. This signifies the presence of volatility clusters and continuity within returns series, with their cumulative sum approaching 1 (0.79), indicating a high level of volatility persistence.

Furthermore, the leverage effect parameter (η_{11}) is positive but lacks statistical significance, implying that both positive and negative shocks exert similar impacts on volatility. Diagnostic evaluations, including the Ljung-Box test, ARCH test, and Nyblom stationary test (refer to Appendix 7), underscored the absence of autocorrelation in the estimated residuals, thereby indicating that the TGARCH (1,1) model proficiently captures the volatility dynamics inherent in the WTI oil returns series.

IV. Conclusion:

In this study, we tried to analyze and model the volatility of WTI oil prices using ARCH-GARCH conditional autoregressive models, in which the study used several important tools and tests to analyze the behavior of the series of returns during the period between 2008 and 2024, which also helped in revealing the characteristics of these returns, in terms of Volatility Clustering, and characterized by a high flattened distribution with heavy tails.

The emperical study concluded by answering the main problem regarding the ability of the GARCH family models, specifically the GARCH, EGARCH, and TGARCH models, to accurately model, analyze, and measure the volatility of West Texas Intermediate crude oil prices. Through the estimation results of the GARCH (1,1) model, which could effectively capture the accumulated volatility in oil prices, the study found that the volatility shocks in West Texas Intermediate crude oil prices are completely permanent. Volatility interacts with market events or shocks, and thus the oil market reacts strongly to new information. By applying the (1,1) EGARCH model, the existence of the leverage effect was revealed, so the negative impact of bad news (negative clashes) in the oil market generates or increases volatility to a greater extent than the positive impact of good news (positive clashes), and that old news in the oil market directly affects volatility. The estimation results of the (1.1) TGARCH model revealed the existence of volatility clusters and continuity in oil prices, that positive and negative shocks have similar effects on volatility, and the results of the (1.1) GARCH model's superiority over other models (1.1) EGARCH and (1.1) TGARCH in extracting oil price volatility as it gives the lowest value for the AIC, SIC, and H-Q criteria.

After examining this topic and discussing its main findings, this study recommends promoting the use of advanced volatility models such as GARCH, TGARCH, and EGARCH in decision-making frameworks to better manage the risks associated with oil price volatility. The models may help in forecasting, stress testing, and strategic planning, as they will provide accurate information to decision-makers and economic policymakers on how to address issues related to oil price volatility and its impact on different economies. Future research should expand its scope by applying these models to other commodities, incorporating macroeconomic factors, and exploring emerging machine-learning techniques for comparative analysis.

Ethical Considerations

This research adheres to established academic and ethical standards in empirical economic and financial research. The study relies exclusively on secondary data obtained from publicly accessible and reputable sources. No human participants, personal data, or confidential information were involved in the research process. All sources are appropriately acknowledged, and the analysis was conducted with transparency, objectivity, and academic integrity.

Author Contributions

All authors contributed substantially to the completion of this study.

- **Hadj Kouider Abdelhadi** conceived the research idea, designed the econometric methodology, and supervised the empirical analysis.
- **Baghafar Abdelkader** contributed to the theoretical framework, literature review, and interpretation of results.
- **Sbai Mhammed** participated in data collection, statistical analysis, and model estimation.
- **Foudou Mohamed** contributed to the discussion of findings, policy implications, and final manuscript revision.

All authors have read and approved the final version of the manuscript.

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Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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Appendices:

Appendix 01: Estimation results of the ARMA(1,1) model

```
Call:
arima(x = oil_ts, order = c(1, 0, 1))

Coefficients:
          ar1     ma1  intercept
      -0.1698  0.4084    -0.0003
s.e.   0.2573  0.2377     0.0043

sigma^2 estimated as 0.002472:  log likelihood = 313.27,  aic = -618.55
```

Appendix 02: GARCH (1,1) model

Estimation results

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002364    0.002487   0.9506 0.341806
omega   0.000293    0.000162   1.8092 0.070426
alpha1  0.519812    0.128467   4.0463 0.000052
beta1   0.479188    0.101366   4.7273 0.000002

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.002364    0.002359   1.0020 0.316356
omega   0.000293    0.000169   1.7357 0.082614
alpha1  0.519812    0.262395   1.9810 0.047588
beta1   0.479188    0.129486   3.7007 0.000215

LogLikelihood : 334.5738
```

Appendix 03: GARCH (1,1) model

diagnostics results

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.3719  0.5420
Lag[2*(p+q)+(p+q)-1][5]    1.9267  0.6358
Lag[4*(p+q)+(p+q)-1][9]    3.1492  0.7341
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.03583 0.500 2.000  0.8499
ARCH Lag[5]   1.21692 1.440 1.667  0.6698
ARCH Lag[7]   1.87716 2.315 1.543  0.7433

Nyblom stability test
------------------------------------
Joint Statistic:  0.5219
Individual Statistics:
mu     0.04481
omega  0.06864
alpha1 0.06445
beta1  0.07485

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.07 1.24 1.6
Individual Statistic:    0.35 0.47 0.75
```

Appendix 04: EGARCH (1,1) model

Estimation results

Appendix 05: EGARCH (1,1) model

diagnostics results

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.003119    0.003707 -0.84133 0.400165
ar1     0.272229    0.288698  0.94295 0.345704
ma1    -0.132007    0.291596 -0.45270 0.650762
omega  -1.271164    0.586111 -2.16881 0.030097
alpha1 -0.308825    0.082959 -3.72261 0.000197
beta1   0.794522    0.093739  8.47592 0.000000
gamma1  0.361025    0.161363  2.23734 0.025264

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.003119     0.00361 -0.86402 0.387577
ar1     0.272229     0.15436  1.76365 0.077791
ma1    -0.132007     0.13920 -0.94834 0.342956
omega  -1.271164     0.68241 -1.86275 0.062498
alpha1 -0.308825     0.13828 -2.23325 0.025533
beta1   0.794522     0.11288  7.03865 0.000000
gamma1  0.361025     0.24638  1.46528 0.142843

LogLikelihood : 340.8231
```

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.005  0.3161
Lag[2*(p+q)+(p+q)-1][5]     5.189  0.1387
Lag[4*(p+q)+(p+q)-1][9]     6.984  0.2005
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.06983 0.500 2.000  0.7916
ARCH Lag[5]   1.16838 1.440 1.667  0.6837
ARCH Lag[7]   2.07743 2.315 1.543  0.7009

Nyblom stability test
------------------------------------
Joint Statistic:  0.9724
Individual Statistics:
mu     0.09721
ar1    0.15078
ma1    0.19569
omega  0.10292
alpha1 0.05437
beta1  0.11557
gamma1 0.20819

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75
```

Appendix 06: TGARCH (1,1) model

Estimation results

Appendix 07: TGARCH (1,1) model

diagnostics results

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : fGARCH(1,1)
fGARCH Sub-Model        : TGARCH
Mean Model      : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.002855    0.003651 -0.78211 0.434152
ar1     0.259340    0.288096  0.90019 0.368022
ma1    -0.134104    0.292670 -0.45821 0.646802
omega   0.012391    0.007642  1.62132 0.104949
alpha1  0.216118    0.108768  1.98696 0.046927
beta1   0.577592    0.154199  3.74575 0.000180
eta11   0.999999    0.655324  1.52596 0.127020

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.002855    0.003381 -0.84446 0.398410
ar1     0.259340    0.141633  1.83106 0.067091
ma1    -0.134104    0.151562 -0.88481 0.376257
omega   0.012391    0.015276  0.81111 0.417300
alpha1  0.216118    0.193702  1.11572 0.264541
beta1   0.577592    0.275409  2.09722 0.035974
eta11   0.999999    1.278246  0.78232 0.434026

LogLikelihood : 341.0925
```

```
Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.9681 0.32515
Lag[2*(p+q)+(p+q)-1][5]    7.6402 0.03617
Lag[4*(p+q)+(p+q)-1][9]    9.3378 0.06921
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.09599 0.500 2.000  0.7567
ARCH Lag[5]   0.78475 1.440 1.667  0.7977
ARCH Lag[7]   1.36608 2.315 1.543  0.8478

Nyblom stability test
------------------------------------
Joint Statistic:  0.9358
Individual Statistics:
mu     0.10822
ar1    0.22473
ma1    0.25972
omega  0.14706
alpha1 0.13204
beta1  0.10525
eta11  0.04795

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75
```